A Universal Look-Ahead Algorithm for Pipelining IIR Filters

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ABSTRACT

Look-ahead (LA) techniques can pipeline IIR digital filters to attain high sampling rate. In the literatures, however, only simple recursive formulas are derived for special cases. In this paper, we present a universal look-ahead (ULA) approach to find the coefficients systematically. This approach is based on that the polynomial multiplication can be expressed in a matrix form. Hence, some matrix operations, such as inversion and multiplication can be applied to calculate the coefficients directly. The proposed approach is suitable for the design of pipelined recursive digital filters with any LA scheme and arbitrary pipelining levels.

I. INTRODUCTION

Look-ahead (LA) techniques are highly effective in attaining high-speed VLSI implementation of recursive digital filters [1]. Several LA approaches have been proposed [2][4]. One is referred to as clustered look-ahead (CLA) algorithm, which clustered the past output data to achieve pipelined IIR filters. Since CLA cannot guarantee to be stable, scattered look-ahead (SLA) algorithm is proposed, which uses equally separated past output data and yields to stable pipelined IIR filters with linear increasing in hardware. Furthermore, distributed look-ahead (DLA) algorithm in [3] combines the two aforementioned schemes to reach stable design with reduced hardware complexity.

In general, an M -stage LA pipelined recursive filter can be obtained by multiplying the numerator and the denominator of the transfer function by an augmented polynomial, $D(z)$. By choosing proper order and coefficients of $D(z)$, we obtain either the M -stage CLA pipelined filter or the M -stage SLA pipelined filter. Most LA researches focus on the hardware reduction and stability of IIR filters. Little attention is paid to design an efficient algorithm to find augmented polynomial coefficients. In [4], a set of simultaneous equations are formulated to solve coefficients in SLA cases. In [3], a recursive expression to find coefficients is given, but it can be only applied to 2nd order IIR filters. For arbitrary number of pipelined stages and different pipelined IIR filter forms, it is hard to derive the general algebraic formula. In this paper, we propose a systematic approach to find the coefficients of the augmented polynomial efficiently with any LA algorithm. This approach is based on that the polynomial multiplication can be expressed in a matrix form. In this manner, matrix inversion and multiplication can be applied to calculate the coefficients directly without recursive formula. Furthermore, if the number of pipelined stages increases, we only have to enlarge the matrix size and use the new matrix to calculate the coefficients directly.

The organization of this paper is as follows. First, we introduce the existing LA schemes in Section II. In Section III, we describe our proposed matrix approach to calculate the augmented polynomial coefficients for pipelining M -stage IIR filter. In Section IV, several examples demonstrate the usage of proposed matrix. Finally, the conclusion is given in Section V.

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II. EXISTING LOOK-AHEAD ALGORITHMS

The transfer function of N th-order recursive filter is described by

$$H(z) = \frac{B(z)}{A(z)} = \frac{\sum_{i=0}^N b_i z^{-i}}{1 + \sum_{i=1}^N a_i z^{-i}}. \quad (1)$$

The LA algorithm finds the augmented polynomial $D(z)$ where

$$D(z) = \sum_{i=0}^L d_i z^{-i} = 1 + \sum_{i=1}^L d_i z^{-i}. \quad (2)$$

Then the pipelined filter is attained by multiplying $D(z)$ to both the denominator and numerator in (1)

$$H_{LA}(z) = \frac{B(z)D(z)}{A(z)D(z)} = \frac{B(z)D(z)}{Q(z)}. \quad (3)$$

For different LA algorithms, the pipelined IIR filter transfer functions $H_{LA}(z)$ are in different forms. Three existing LA algorithms are summarized here.

A. Clustered Look-ahead (CLA) Algorithm

For the M -stage CLA pipelined IIR filters, the denominator of the transfer function can be expressed in the form of

$$Q_{CLA,M}(z) = 1 + z^{-M} \sum_{i=0}^{N-1} q_{i+1} z^{-i}, \quad (4)$$

where M is the pipelined stage, q_i is the coefficient of pipelined filter. Fig. 1(a) shows the architecture of CLA pipelined IIR filters. The output data $y(n)$ can be described by the cluster of N past data $y(n-M)$, $y(n-M-1)$, $\dots$, $y(n-M-N+1)$. In [3], the augmented polynomial coefficients d_i can be found by iterative calculating as follows,

$$d_0 = 1, \quad d_1 = -a_1, \quad d_j = -\sum_{i=1}^j a_i d_{j-i}. \quad (5)$$

B. Scattered Look-ahead (SLA) Algorithm

For the M -stage SLA pipelined IIR filters, the denominator of the transfer function is obtained as

$$Q_{SLA,M}(z) = 1 + \sum_{i=1}^N q_{Mi} z^{-iM}. \quad (6)$$

The denominator of the resulted transfer function contains N scattered terms, z^{-M} , z^{-2M} , $\dots$, z^{-NM} . There are $N(M-1)$ coefficients of augmented polynomial d_i to force as many as coefficients in the denominator to zero. The architecture is shown in Fig. 1(b).

In [4], the coefficients d_i can be attained by solve $N(M-1)$ linear simultaneous equations

Fig. 1. LA pipelined IIR filters: (a) CLA realization and (b) SLA realization (c) DLA realization

$$d_1 = -a_1, \quad d_i = -\sum_{j=1}^{i-1} a_j d_{i-j} - a_i, \quad (7)$$

whrer $i = 2, \dots, M-1, M+1, \dots, tM-1, tM+1, \dots, NM-1$.

C. Distributed Look-ahead (DLA) Algorithm

In [3], the transfer function of M -stage DLA pipelined IIR filters is written as,

$$H_{DLA,M}(z) = \frac{B(z) \cdot (1 + \sum_{i=1}^P d_i z^{-i})}{1 + z^{-M} (q_1 + q_2 z^{-k_1} + q_3 z^{-(k_1+k_2)} + \dots + q_N z^{-(k_1+k_2+\dots+k_L)})}, \quad (8)$$

where $P = M + (k_1 + k_2 + \dots + k_L) - N$. The architecture is shown in Fig. 1(c). DLA sees CLA and SLA as special cases by substituting k_i with different values. The coefficients of augmented polynomial can not be described by a general formula. In [3], only a 2nd-order pipelined IIR filter is given.

III. PROPOSED MATRIX APPROACH

In this section, a systematic matrix approach is presented to find the augmented polynomial coefficients for any LA algorithm.

Substitute (1) and (2) into (3), then the $Q(z)$ is written as,

$$Q(z) = 1 + \sum_{i=1}^{N+M-1} q_i z^{-i} = (1 + \sum_{i=1}^N a_i z^{-i}) (1 + \sum_{i=1}^{M-1} d_i z^{-i}), \quad (9)$$

where the coefficients q_i is

$$q_i = \sum_{j=0}^{M-1} a_{i-j} d_j, \quad \begin{cases} a_0 = d_0 = 1 \\ a_i = 0, \quad \text{for } i < 0 \text{ or } i = N+1 \sim N+M-1 \end{cases} \quad (10)$$

By employing (11), we can rewrite (10) into the matrix form

$$\mathbf{A}\mathbf{D} = \mathbf{Q}, \quad (11)$$

where $\mathbf{A}$ denotes the $(N+M)$ -by- M matrix, $\mathbf{D}$ denotes the M -by-1 column vector and $\mathbf{Q}$ denotes $(N+M)$ -by-1 column vector:

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & \dots & \dots & 0 \\ a_1 & 1 & \ddots & & \vdots \\ a_2 & a_1 & 1 & \ddots & \vdots \\ \vdots & a_2 & a_1 & \ddots & 0 \\ a_N & \vdots & a_2 & \ddots & 1 \\ 0 & a_N & \vdots & \ddots & a_1 \\ \vdots & \ddots & a_N & \ddots & a_2 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \dots & \dots & 0 & a_N \end{bmatrix}, \quad \mathbf{D} = \begin{bmatrix} 1 \\ d_1 \\ d_2 \\ \vdots \\ d_{M-1} \end{bmatrix}, \quad \mathbf{Q} = \begin{bmatrix} 1 \\ q_1 \\ q_2 \\ \vdots \\ q_{N+M-1} \end{bmatrix}. \quad (12)$$

That is, we have $(M-1)$ variables, d_1 to d_{M-1} , are determined to let the $(M-1)$ values of q_i among total $(N+M-1)$ coefficients in the denominator equal to zero. Therefore, we extract the corresponding rows with $q_i = 0$ to solve d_i as follows,

$$\begin{bmatrix} \underline{e}_1 \\ k_1 \text{ row} \\ k_2 \text{ row} \\ \vdots \\ k_{M-1} \text{ row} \end{bmatrix} \begin{bmatrix} 1 \\ d_1 \\ d_2 \\ \vdots \\ d_{M-1} \end{bmatrix} = \begin{bmatrix} 1 \\ q_{k_1} \\ q_{k_2} \\ \vdots \\ q_{k_{M-1}} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \quad (13)$$

where $\underline{e}_1$ is the first row of $\mathbf{A}$, which is $[1, 0, 0, \dots, 0]$, and k_i is the row index that designers can choose the corresponding row vectors from $\mathbf{A}$. Hence, the value of k_i can be chosen among 1 to $N+M-1$. Written (13) in the simple form,

$$\mathbf{K}\mathbf{D} = \mathbf{Q}_d, \quad (14)$$

where $\mathbf{K} = [\underline{e}_1, (k_1 \text{ row})^T, (k_2 \text{ row})^T, \dots, (k_{M-1} \text{ row})^T]^T$, and $\mathbf{Q}_d = [1, 0, 0, \dots, 0]^T$. The superscript T denotes transpose, and $\mathbf{D}$ is the column vector composed of coefficients of augmented polynomial. We deduce $\mathbf{D}$ as the inverse of $\mathbf{K}$ multiplies $\mathbf{Q}_d$ as follows,

$$\mathbf{D} = \mathbf{K}^{-1} \mathbf{Q}_d. \quad (15)$$

Note that the solution for the vector $\mathbf{D}$ is simply the first column of the inverse of $\mathbf{K}$. After finding the vector $\mathbf{D}$, for the remaining non-zero coefficients in the denominator, using the remaining rows of $\mathbf{A}$ times the vector $\mathbf{D}$, that is

$$\begin{bmatrix} r_1 \text{ row} \\ r_2 \text{ row} \\ \vdots \\ r_N \text{ row} \end{bmatrix} \begin{bmatrix} 1 \\ d_1 \\ d_2 \\ \vdots \\ d_{M-1} \end{bmatrix} = \begin{bmatrix} q_{r_1} \\ q_{r_2} \\ \vdots \\ q_{r_N} \end{bmatrix} = \begin{bmatrix} W_1 \\ W_2 \\ \vdots \\ W_N \end{bmatrix}, \quad (16)$$

where r_i is the remaining rows with $q_i \neq 0$, and W_i is the non-zero values in the denominator of pipelined transfer function. Similarly, (16) is written in short form,

$$\mathbf{R}\mathbf{D} = \mathbf{Q}_w, \quad (17)$$

where $\mathbf{R} = [(r_1 \text{ row})^T, (r_2 \text{ row})^T, \dots, (r_N \text{ row})^T]^T$, and $\mathbf{Q}_w = [W_1, W_2, W_3, \dots, W_N]^T$.

This is the matrix approach to find the coefficients of pipelined IIR filter in systematic and efficient procedure. For different pipelined stages or any kind of LA algorithm, we only have to formulate the new matrix $\mathbf{K}$ and find its inversion to get the column vector $\mathbf{D}$. Then multiply $\mathbf{D}$ by the matrix $\mathbf{R}$, the coefficients in the denominator are also obtained. For the special cases, CLA and SLA, the resulted matrix approach is shown below.

A. Clustered Look-ahead (CLA)

Consider M -stage pipelined approach, from (11) and (12), we formulate

$$\begin{bmatrix} 1 & 0 & \dots & \dots & 0 \\ a_1 & 1 & \ddots & & \vdots \\ a_2 & a_1 & 1 & \ddots & \vdots \\ \vdots & a_2 & a_1 & \ddots & 0 \\ a_N & \vdots & a_2 & \ddots & 1 \\ 0 & a_N & \vdots & \ddots & a_1 \\ \vdots & \ddots & a_N & \ddots & a_2 \\ \vdots & & \ddots & \ddots & \vdots \\ 0 & \dots & \dots & 0 & a_N \end{bmatrix} \begin{bmatrix} 1 \\ d_1 \\ d_2 \\ d_3 \\ \vdots \\ d_{M-1} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ W_1 \\ W_2 \\ \vdots \\ W_N \end{bmatrix}, \quad (18)$$

It can be seen the non-zero coefficients in the $\mathbf{Q}$ vector clustered in the last N positions. Then we use (15) to solve $\mathbf{D}$

$$\mathbf{D} = \begin{bmatrix} 1 \\ d_1 \\ d_2 \\ \vdots \\ d_{M-1} \end{bmatrix} = \begin{bmatrix} \frac{e_1}{2^{nd} \text{ row}} \\ 3^{rd} \text{ row} \\ \vdots \\ M^{th} \text{ row} \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \quad (19)$$

Note $\mathbf{D}$ is the first column of $\mathbf{K}^{-1}$. Finally, we use (17) to find non-zero values in the denominator, W_1 to W_N , as the following equation

$$\begin{bmatrix} (M+1)^{th} \text{ row} \\ (M+2)^{th} \text{ row} \\ \vdots \\ (M+N)^{th} \text{ row} \end{bmatrix} \begin{bmatrix} 1 \\ d_1 \\ d_2 \\ \vdots \\ d_{M-1} \end{bmatrix} = \begin{bmatrix} q_M \\ q_{M+1} \\ q_{M+2} \\ \vdots \\ q_{M+N-1} \end{bmatrix} = \begin{bmatrix} W_1 \\ W_2 \\ W_3 \\ \vdots \\ W_N \end{bmatrix}, \quad (20)$$

B. Scattered Look-ahead (SLA)

For pipelining M -stage SLA IIR filters, (11) becomes

$$\begin{bmatrix} 1 & 0 & \dots & \dots & 0 \\ a_1 & 1 & \ddots & & \vdots \\ a_2 & a_1 & 1 & \ddots & \vdots \\ \vdots & a_2 & a_1 & \ddots & 0 \\ a_N & \vdots & a_2 & \ddots & 1 \\ 0 & a_N & \vdots & \ddots & a_1 \\ \vdots & \ddots & a_N & \ddots & a_2 \\ \vdots & & \ddots & \ddots & \vdots \\ 0 & \dots & \dots & 0 & a_N \end{bmatrix} \begin{bmatrix} 1 \\ d_1 \\ d_2 \\ d_3 \\ \vdots \\ d_{N(M-1)} \end{bmatrix} = \begin{bmatrix} 1 \\ \mathbf{z} \\ W_1 \\ \mathbf{z} \\ W_2 \\ \vdots \\ \mathbf{z} \\ W_N \end{bmatrix}, \quad (21)$$

where $\mathbf{z}$ is the $(M-1)$ -by-1 column vector with all elements being zero. Note the non-zero terms in the $\mathbf{Q}$ vector scattered with equally distance M . Similarly, from (15), we can deduce $\mathbf{D}$

$$\mathbf{D} = \begin{bmatrix} 1 \\ d_1 \\ d_2 \\ d_3 \\ \vdots \\ d_{N(M-1)} \end{bmatrix} = \begin{bmatrix} \frac{e_1}{2^{nd} \text{ row}} \\ \vdots \\ M^{th} \text{ row} \\ \vdots \\ (2M+2)^{th} \text{ row} \\ \vdots \\ NM^{th} \text{ row} \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}. \quad (22)$$

Then, the coefficients in the denominator are found by using (17)

$$\begin{bmatrix} (M+1)^{th} \text{ row} \\ (2M+1)^{th} \text{ row} \\ (3M+1)^{th} \text{ row} \\ \vdots \\ (NM+1)^{th} \text{ row} \end{bmatrix} \begin{bmatrix} 1 \\ d_1 \\ d_2 \\ \vdots \\ d_{N(M-1)} \end{bmatrix} = \begin{bmatrix} q_M \\ q_{2M} \\ q_{3M} \\ \vdots \\ q_{NM} \end{bmatrix} = \begin{bmatrix} W_1 \\ W_2 \\ W_3 \\ \vdots \\ W_N \end{bmatrix}, \quad (23)$$

From the above two cases, we find both the desired $\mathbf{D}$ and $\mathbf{Q}_W$ vector in a systematic procedure. In fact, we can choose one of the LA algorithms with arbitrary pipelined stages and use the proposed matrix approach to find coefficients.

In summary, the procedure to find the coefficients for pipelining IIR filters can be divided into 4 steps:

- 1) Determine the number of pipelined stage.
- 2) Formulate the matrix $\mathbf{A}$ based on the coefficients in the denominator of the original transfer function.
- 3) Choose one of the LA algorithms, and solve the augmented polynomial coefficients vector $\mathbf{D}$ by using (15)
- 4) Calculate the pipelined IIR filter coefficients vector $\mathbf{Q}_W$ by using (17)

IV. PRACTICAL DESIGN EXAMPLES

In this section, we give some examples to show how to find coefficients with matrix approach. Consider the transfer function obtained from [5],

$$H(z) = \frac{1}{1 - 1.25z^{-1} + 0.375z^{-2}}. \quad (24)$$

Three pipelined IIR filter using different algorithm are presented.

A. Clustered Look-ahead (CLA)

For a 6-stage CLA pipelined digital filter, use (19) to solve $\mathbf{D}$

$$\mathbf{D} = \begin{bmatrix} 1 \\ d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ -1.25 & 1 & 0 & 0 & 0 & 0 \\ 0.375 & -1.25 & 1 & 0 & 0 & 0 \\ 0 & 0.375 & -1.25 & 1 & 0 & 0 \\ 0 & 0 & 0.375 & -1.25 & 1 & 0 \\ 0 & 0 & 0 & 0.375 & -1.25 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1.25 \\ 1.1875 \\ 1.0156 \\ 0.8242 \\ 0.6494 \end{bmatrix}. \quad (25)$$

Then we find W_1 and W_2 using (20)

$$\mathbf{Q}_W = \mathbf{R}\mathbf{D} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0.375 & -1.25 \\ 0 & 0 & 0 & 0 & 0 & 0.375 \end{bmatrix} \begin{bmatrix} 1 \\ 1.25 \\ 1.1875 \\ 1.0156 \\ 0.8242 \\ 0.6494 \end{bmatrix} = \begin{bmatrix} W_1 \\ W_2 \end{bmatrix} = \begin{bmatrix} -0.5027 \\ 0.2435 \end{bmatrix}. \quad (26)$$

The pipelined IIR filter transfer function becomes

$$H(z) = \frac{1 + 1.25z^{-1} + 1.1875z^{-2} + 1.0156z^{-3} + 0.8242z^{-4} + 0.6494z^{-5}}{1 - 0.5027z^{-6} + 0.2435z^{-7}}. \quad (27)$$

Fig.2 shows the architecture of the CLA pipelined IIR filter.

Fig. 2. Implementation of the 6-stage CLA pipelined IIR filter

Fig. 3. Implementation of the 3-stage SLA pipelined IIR filter

Fig. 4. Implementation of the pipelined IIR filter in arbitrary pipelining levels

B. Scattered Look-ahead (SLA)

To obtain 3-stage pipelined SLA filter, use (22) to calculate $\mathbf{D}$

$$\mathbf{D} = \begin{bmatrix} 1 \\ d_1 \\ d_2 \\ d_3 \\ d_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -1.25 & 1 & 0 & 0 & 0 \\ 0.375 & -1.25 & 1 & 0 & 0 \\ 0 & 0 & 0.375 & -1.25 & 1 \\ 0 & 0 & 0 & 0.375 & -1.25 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1.25 \\ 1.1875 \\ 0.4688 \\ 0.1406 \end{bmatrix}. \quad (28)$$

Then use (23) to find W_1 and W_2

$$\mathbf{RD} = \begin{bmatrix} 0 & 0.375 & -1.25 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0.375 \end{bmatrix} \begin{bmatrix} 1 \\ 1.25 \\ 1.1875 \\ 0.4688 \\ 0.1406 \end{bmatrix} = \begin{bmatrix} W_1 \\ W_2 \end{bmatrix} = \begin{bmatrix} -0.5469 \\ 0.0527 \end{bmatrix}. \quad (29)$$

The transfer function becomes,

$$H(z) = \frac{1 + 1.25z^{-1} + 1.1875z^{-2} + 0.4688z^{-3} + 0.1406z^{-4}}{1 - 0.5469z^{-3} + 0.0527z^{-6}}. \quad (30)$$

The architecture of the 3-stage SLA pipelined IIR filter shows in Fig. 3.

C. General Cases

For a general design example, we arbitrarily determine the denominator to be $1 + q_4z^{-4} + q_6z^{-6}$, (11) is formulated as

$$\mathbf{AD} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -1.25 & 1 & 0 & 0 & 0 \\ 0.375 & -1.25 & 1 & 0 & 0 \\ 0 & 0.375 & -1.25 & 1 & 0 \\ 0 & 0 & 0.375 & -1.25 & 1 \\ 0 & 0 & 0 & 0.375 & -1.25 \\ 0 & 0 & 0 & 0 & 0.375 \end{bmatrix} \begin{bmatrix} 1 \\ d_1 \\ d_2 \\ d_3 \\ d_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}. \quad (31)$$

Use (16) to solve $\mathbf{D}$

$$\mathbf{D} = \begin{bmatrix} 1 \\ d_1 \\ d_2 \\ d_3 \\ d_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -1.25 & 1 & 0 & 0 & 0 \\ 0.375 & -1.25 & 1 & 0 & 0 \\ 0 & 0.375 & -1.25 & 1 & 0 \\ 0 & 0 & 0.375 & -1.25 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1.25 \\ 1.1875 \\ 1.0156 \\ 0.3047 \end{bmatrix}. \quad (32)$$

Then use (17) to solve $\mathbf{Q}_w$

$$\mathbf{Q}_w = \begin{bmatrix} W_1 \\ W_2 \end{bmatrix} = \mathbf{RD} = \begin{bmatrix} 0 & 0 & 0.375 & -1.25 & 1 \\ 0 & 0 & 0 & 0 & 0.375 \end{bmatrix} \begin{bmatrix} 1 \\ 1.25 \\ 1.1875 \\ 1.0156 \\ 0.3047 \end{bmatrix} = \begin{bmatrix} -0.5195 \\ 0.1143 \end{bmatrix}. \quad (33)$$

The pipelined transfer function has the following form,

$$H(z) = \frac{1 + 1.25z^{-1} + 1.1875z^{-2} + 1.0156z^{-3} + 0.3047z^{-4}}{1 - 0.5195z^{-4} + 0.1143z^{-6}}. \quad (34)$$

The implementation shows in Fig. 4.

V. CONCLUSION

In this paper, we propose ULA algorithm to systematically find the coefficients for pipelining IIR filters. Based on our approach, the design of pipelined recursive digital filters with any LA scheme and arbitrary pipelining levels can be derived efficiently.

REFERENCES

- [1] Gu, Y. and K. K. Parhi, "High-Speed Architecture Design of Tomlinson-Harashima Precoders," *IEEE Trans. Circuits and Systems Part-I: Fundamental Theory and Applications*, vol. 54, no. 10, pp. 1929-1937, Sept. 2007
- [2] K. K. Parhi and D. G. Messerschmitt, "Pipeline interleaving and parallelism in recursive digital filters-Part I: Pipelining using scattered look-ahead and decomposition," *IEEE Trans. Acoustics, Speech, Signal Processing*, vol. 37, pp. 1099-1117, July 1989.
- [3] A. K. Shaw and M. Imtiaz, "A general Look-Ahead algorithm for pipelining IIR filters," in *Proc. IEEE ISCAS*, 1996, pp. 237-240.
- [4] Y. C. Lim, "A new approach for deriving scattered coefficients of pipelined IIR filters," *IEEE Trans. Signal Processing*, vol. 43, pp. 2405-2406, 1995.
- [5] K. K. Parhi, *VLSI Digital Signal Processing System Design and Implementation*, John Wiley & Son, Inc., New York, 1999.