Low-Complexity Two-Step Optimization in Active-IRS-Assisted Uplink NOMA Communication

Chi-Wei Chen, Wen-Chiao Tsai, and An-Yeu Wu, Fellow, IEEE

Abstract—Recently, the active intelligent reflecting surface (IRS) has been proposed to adjust the phase and amplify the magnitude of the incident signal simultaneously. It has a more reasonable hardware overhead compared with the conventional relay. This letter aims to maximize the sum rate of multiple users in the active-IRS-assisted uplink non-orthogonal multiple access (NOMA) system. We propose a low-complexity two-step optimization algorithm to decompose the original non-convex problem into two sub-problems. First, the extreme low-complexity fixed point iteration (FPI) method is proposed to optimize the phase shifts. Then, two algorithms are proposed to solve the amplification optimization problem: the convergence-guaranteed quadratic transform (QT) and the low-complexity generalized eigenvalue decomposition (GEVD) algorithms. Simulation results show that the performance can be enhanced significantly compared with the orthogonal multiple access (OMA) and the passive-IRS scheme.

Index Terms—Active-intelligent reflecting surface (Active-IRS), non-orthogonal multiple access (NOMA), sum rate optimization, uplink.

I. INTRODUCTION

To fulfill the explosive demand of data transmission, intelligent reflecting surface (IRS) composed of a large number of passive reflecting elements has been considered as the potential technique for 6G communication due to its capability to reconfigure the wireless environment [1]. Most works study the reflecting element implemented by the unit-modulus phase shifters without signal amplification, called passive-IRS. Moreover, the above works allocate radio resources to users on the orthogonal multiple access (OMA) scheme as shown in Fig.1(a), which incurs spectral inefficiency in massive connectivity of future Internet of Things (IoT) applications.

To further improve the spectral efficiency, the passive-IRS-assisted non-orthogonal multiple access (NOMA) systems have shown efficiency owing to the same resource block utilization in [2], [3], [4], and [5]. In [2], the authors consider the downlink communication to maximize the minimum decoding signal-to-interference-plus-noise ratio (SINR) of all users by jointly optimizing the BS’s power allocation and the IRS’s phase shifts. In [3], it shows that the sum rate for the uplink system is independent of the decoding order. Both these works transform the original non-convex problem to a convex semidefinite relaxation (SDR) problem. Moreover, in [4], the successive convex approximation (SCA) technique and sequential rank-one constraint relaxation (SROCR) approach are proposed for efficient optimization in the ideal and non-ideal IRS scheme, respectively. Furthermore, the physical layer security application in the IRS-NOMA system is discussed in [5]. Although the performance has improved by NOMA and passive-IRS techniques, the unaffordable high complexity of SDR and the double fading effect of passive-IRS still limit the practical applications.

To compensate the drawback of double fading effect in passive-IRS, the active-IRS has been recently proposed. The active-IRS is different from the conventional amplify-and-forward (AF) relay that contains power-hungry radio frequency (RF) chains to receive the incident signal and transmit it with the linear beamforming matrix. It only comprises the negative resistance components, such as tunnel diodes, to reflect and amplify the incident signal instantaneously without the sensing capability [6], [7], [8]. Due to the RF chain-free architecture of active-IRS, the power consumption and hardware cost can be largely reduced. In [6], the circuit of active-IRS has been designed and measured to validate the signal model. The precoder and active-IRS are jointly optimized to maximize the spectral efficiency for multi-users communication systems in [6] and [7]. All these works show that the active-IRS can provide better performance and efficiency compared with passive-IRS.

In this letter, we aim to exploit the signal amplify capability of the active-IRS in a NOMA system to further boost the performance in IoT applications. We propose a low-complexity two-step optimization to maximize the sum rate. First, the extreme low-complexity fixed point iteration (FPI) method is proposed to optimize the phase, achieving 4000 times faster than SDR. Then, the convergence-guaranteed quadratic transform (QT) and the low-complexity generalized eigenvalue decomposition (GEVD) algorithms are proposed to solve the amplification optimization problem. Furthermore, the high complexity of SDR and the double fading effect of passive-IRS still limit the practical applications.

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flexibility of two-step optimization can regard the passive-IRS scheme as special case by processing the first step only.

II. SYSTEM MODEL AND PROBLEM FORMULATION

A. System Model

We consider a $K$ users SISO uplink system with a single antenna at the BS and UE, as shown in Fig. 1(b). Assume that direct link is blocked and can be ignored due to the unfavorable propagation conditions [3], the active-IRS equipped with $M$ active reflecting elements is deployed to enhance the NOMA communication performance. In the transmission, the received signal at the BS can be expressed as

$$s = g^H P_a \Theta \sum_{k=1}^{K} h_k \sqrt{p_k} s_k + g^H P_a \Theta n_a + n.$$  

(1)

$s_k$ denotes the transmitted signal with the unit power of $\mathbb{E}[|s_k|^2] = 1, \forall k \in K = \{1, 2, \ldots, K\}$ and $p_k$ denotes the transmitted power in each UE device. Let $n_a \in \mathbb{C}^{M \times 1} \sim \mathcal{CN}(0, \sigma^2_n I_M)$ and $n \in \mathbb{C} \sim \mathcal{CN}(0, \sigma^2_n)$ be the dynamic and static additive white Gaussian noise at IRS and BS, respectively, where $\sigma^2_n$ and $\sigma^2_n$ are the corresponding noise variance.

Assume that the perfect channel state information (CSI) is available at the IRS controller, $g \in \mathbb{C}^{M \times 1}$ and $h_k \in \mathbb{C}^{M \times 1}$ are the BS and the k-th UE-IRS channels, respectively. The active-IRS is composed of the diagonal amplification matrix $P_a = \text{diag}([p_{a,1}, p_{a,2}, \ldots, p_{a,M}]) \in \mathbb{R}^{M \times M}$ and the diagonal phase shift matrix $\Theta = \text{diag}([e^{j\theta_1}, e^{j\theta_2}, \ldots, e^{j\theta_M}]) \in \mathbb{C}^{M \times M}$, where $\theta_m \in [0, 2\pi), \forall m \in M = \{1, 2, \ldots, M\}$.

To mitigate the user-interference in (1), the BS processes the successive interference cancellation (SIC). We will show that the decoding order will not affect the sum rate in the active-IRS system as in the passive-IRS scheme [3]. Firstly, the SINR of the k-th UE is written as

$$\gamma_k = \frac{|g^H P_a \Theta h_k|^2 p_k}{\sum_{i=k+1}^{K} (|g^H P_a \Theta h_i|^2 p_i + \sigma^2_n |g^H P_a \Theta|^2 + \sigma^2_n)}.$$  

(2)

Then, the sum rate of all users can be expressed as

$$R_{\text{sum}} = \sum_{k=1}^{K} \log_2 (1 + \gamma_k)$$  

(a) $\log_2 (1 + \sum_{k=1}^{K} \frac{|g^H P_a \Theta h_k|^2 p_k}{\sigma^2_n |g^H P_a \Theta|^2 + \sigma^2_n})$.

(3)

where (a) holds due to the telescoping product expression forms [3]. Therefore, the sum rate is order-invariant.

B. Problem Formulation

We aim to jointly optimize $P_a, \Theta, \{p_k\}$ to maximize the sum rate. Hence, the problem can be formulated as

$$\max_{P_a, \Theta, \{p_k\}} \log_2 (1 + \sum_{k=1}^{K} \frac{|g^H P_a \Theta h_k|^2 p_k}{\sigma^2_n |g^H P_a \Theta|^2 + \sigma^2_n})$$

(4a)

subject to $|\Theta_{m,n}| = 1, \forall m \in M$,  

$$\sum_{k=1}^{K} \|P_a \Theta h_k\|^2 + \sigma^2_n \|P_a \Theta\|^2 \leq p_{a,\text{max}}^2,$$

(4b)

$$p_k \leq p_{k,\text{max}}, \forall k \in K,$$

(4c)

where (4b) is the unit-modulus constraint of the phase shifter. Moreover, due to the hardware limitation of the reflecting elements and the limited power resource, the power of the reflected signal at the active-IRS planar should not be greater than the maximal power allowance $p_{a,\text{max}}^2$, which is constrained by (4c). $p_{k,\text{max}}$ is the maximal transmit power at the k-th UE, and we has the following lemma.

Lemma 1: Each user can transmit the signal at the maximal power, i.e., $p_k = p_{k,\text{max}}, \forall k \in K$, to maximize sum rate (4a).

Please refer to Appendix for the detailed proof of the Lemma 1 by contradiction. Define $v = [e^{j\theta_1}, e^{j\theta_2}, \ldots, e^{j\theta_M}]^T$ as the diagonal vector of $\Theta^H$ and $v_a = P_a v$. Due to the diagonal property in both $P_a$ and $\Theta$, we can obtain

$$|g^H P_a \Theta h_k|^2 = |v_a^H h_k|^2,$$

(5)

where $h_k = \text{diag}(g^H) h_k$ is the k-th cascaded channel. Since $\log_2$ is a monotonic increasing function, the problem can be reformulated as

$$\max_{v_a} v_a^H H_1 v_a$$

(6a)

subject to $v_a^H H_2 v_a \leq p_{a,\text{max}}^2$,  

(6b)

where $H_1 = \sum_{k=1}^{K} p_{k,\text{max}} h_k h_k^H \in \mathbb{C}^{M \times M}$ is a positive definite (PSD) matrix, and $H_2 = \sigma^2_n \text{diag}(g^H) \text{diag}(g) \in \mathbb{R}^{M \times M}$, $H_3 = \sum_{k=1}^{K} p_{k,\text{max}} \text{diag}(h_k) \text{diag}(h_k^H) + \sigma^2_n I_M \in \mathbb{R}^{M \times M}$ are diagonal matrices.

III. TWO-STEP OPTIMIZATION

The quadratic fractional programming problem in (6) is intractable because the non-convex objective (6a) has convex quadratic functions in both numerator and denominator. Therefore, we decompose the problem into two sub-problems, namely, the phase optimization followed by the amplification optimization. Then, we propose a one-shot two-step method instead of alternating optimization because the phase and the amplification are low-correlated, which results in fast convergence. Moreover, the high flexibility of two-step optimization can also be applied to passive-IRS by processing the first step only. The detailed derivation is as follows.

A. Step 1: Optimization of Phase $v$

We first investigate the configurations of the phase shifter as the passive-IRS in [3]. Due to the real and diagonal properties of $H_2$ and $H_3$, the phase vector $v$ is invariant to $v_a^H H_2 v_a$ and $v_a^H H_3 v_a$ when $P_a$ is fixed. Hence, the problem can be simplified as

$$\max_{v_a} v_a^H H_1 v_a$$

(7a)

subject to $|v_m| = 1, \forall m \in M$,  

(7b)

where $H_1 = P_a^H H_1 P_a$.

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The quadratic objective with non-convex unit-modulus constraint optimization problem (7) has been solved by the SDR approach in [3]. By introducing the auxiliary variable \( \mathbf{V} = \mathbf{v} \mathbf{v}^H \) with rank-one constraint, (7) can be equivalent to a semidefinite programming (SDP) problem. Then, dropping the rank-one constraint as relaxation, SDR can obtain the optimal solution \( \mathbf{V} \). Next, the rank-one approximation and the phase projection are applied to derive the sub-optimal \( \mathbf{v} \) from \( \mathbf{V} \). Although SDR is an one-shot optimization without iterations, it incurs high complexity of \( \mathcal{O}(M^{4.5}) \) which is prohibitive in practical implementation with large \( M \).

Therefore, we exploit the extreme low-complexity fixed point iteration (FPI) method to solve the problem (7) as the following lemma [11].

**Lemma 2:** A limit point of the following fixed point iteration is a locally optimal solution of (7):

\[
\mathbf{v}^1 = e^{j\lambda} \mathbf{H}_1 \mathbf{v},
\]

where \( \mathbf{v}^1 \) means the updated phase vector. Please refer to [11] for detailed proof of the Lemma 2. The iteratively updated \( \mathbf{v} \) will make (7a) monotonically converge to a local optimal. We set a pre-define threshold \( \| \mathbf{H}_1 \mathbf{v} \|_1 \leq \epsilon \), the performance saturates and the first step phase optimization is completed.

### B. Step 2: Optimization of Amplification \( \mathbf{p}_a \)

With the fixed local optimal phase shifters, problem (6) becomes a power control problem to allocate the power at active-IRS. Denoting \( \mathbf{p}_a \) as the diagonal vector of \( \mathbf{P}_a \), we can formulate the active-IRS power control problem as

\[
\begin{align*}
\text{maximize} & \quad \frac{\mathbf{p}_a^H \mathbf{\hat{H}}_3 \mathbf{p}_a}{\mathbf{p}_a^H \mathbf{\hat{H}}_2 \mathbf{p}_a + \sigma_n^2} \\
\text{subject to} & \quad \mathbf{p}_a^H \mathbf{\hat{H}}_1 \mathbf{p}_a \leq \mathbf{p}_a^{max},
\end{align*}
\]

where \( \mathbf{\hat{H}}_i = \mathbf{\Theta} \mathbf{H}_i \mathbf{\Theta}^H, i = 1, 2, 3 \). The problem (9) is hard to optimize and cannot be solved by the classical Dinkelbach method directly because the non-convex objective has convex quadratic functions in both numerator and denominator.

Therefore, we proposed two algorithms to tackle this problem. First, we transform the problem (9) to a convex problem via the quadratic transform (QT) method [12], which is convergence-guaranteed. On the other hand, to reduce complexity, we optimize the upper bound of (9a) by the generalized EVD (GEVD) method, which is low-complexity due to the closed-form solution.

1) **Convergence-Guaranteed Quadratic Transform (QT):**

Denote \( \mathbf{\hat{H}}_1 = \mathbf{\hat{U}}_1 \mathbf{\hat{H}}_1 \mathbf{\hat{U}}_1^H \) as the EVD of \( \mathbf{\hat{H}}_1 \), where all the diagonal elements in \( \mathbf{\hat{H}}_1 \) are non-negative real numbers since \( \mathbf{\hat{H}}_1 \) is PSD. Then, (9a) can be rewritten as

\[
\mathbf{p}_a^H \mathbf{\hat{U}}_1 \mathbf{\hat{H}}_1 \mathbf{\hat{U}}_1^H (\mathbf{p}_a^H \mathbf{\hat{H}}_2 \mathbf{p}_a \mathbf{I}_M + \sigma_n^2 \mathbf{I}_M)^{-1} \mathbf{\hat{U}}_1^H \mathbf{p}_a,
\]

which is the multidimensional fractional programming case in [12]. According to [12, Th.2], the quadratic transform can be applied to equivalent the problem (9) as

\[
\begin{align*}
\text{maximize} & \quad f(\mathbf{p}_a, y) \\
\text{subject to} & \quad \mathbf{p}_a^H \mathbf{\hat{H}}_3 \mathbf{p}_a \leq \mathbf{p}_a^{max},
\end{align*}
\]

where \( y \) is the auxiliary vector, and the quadratic function \( f(\mathbf{p}_a, y) \) can be expressed as

\[
2\Re(\mathbf{y}^H \mathbf{\hat{A}}_1 \mathbf{\hat{U}}_1^H \mathbf{p}_a) - \mathbf{y}^H (\mathbf{p}_a^H \mathbf{\hat{H}}_2 \mathbf{p}_a \mathbf{I}_M + \sigma_n^2 \mathbf{I}_M) \mathbf{y}.
\]

Then we can alternately optimize the two quadratic convex vectors, i.e., \( \mathbf{p}_a, y \), separately until the sum rate converges. Please refer to [12] for the detailed proof of the equivalent problem transformation and the strong convergence.

With fixed \( \mathbf{p}_a \), the unconstrained convex problem can be optimally solved by setting \( \frac{\partial f(\mathbf{p}_a, y)}{\partial y} = 0 \), and we can obtain the optimal closed-form

\[
y = (\mathbf{p}_a^H \mathbf{\hat{H}}_2 \mathbf{p}_a \mathbf{I}_M + \sigma_n^2 \mathbf{I}_M)^{-1} \mathbf{\hat{A}}_1 \mathbf{\hat{U}}_1^H \mathbf{p}_a.
\]

With fixed \( y \), the problem becomes a standard quadratic constraint quadratic programming (QCQP) problem. Therefore, \( \mathbf{p}_a \) can be optimally solved by the existing optimization methods such as the alternating direction method of multiplier (ADMM) [13] via the CVX solver tool.

2) **Low-Complexity Generalized EVD (GEVD):**

To further reduce the computational complexity, we aim to derive and optimize the upper bound of (9a). First, we find that

\[
\frac{\mathbf{p}_a^H \mathbf{\hat{H}}_1 \mathbf{p}_a}{\mathbf{p}_a^H \mathbf{\hat{H}}_2 \mathbf{p}_a + \sigma_n^2} \leq \frac{\mathbf{p}_a^H \mathbf{\hat{H}}_2 \mathbf{p}_a + \sigma_n^2}{\sigma_n^2},
\]

and the bound is tight when \( \mathbf{p}_a^H \mathbf{\hat{H}}_2 \mathbf{p}_a \leq \sigma_n^2 \). Due to the huge path loss leading to the weak channel effect, the upper bound optimization shows efficiency in most scenarios, which will be validated in the numerical simulations. Furthermore, because \( \frac{\mathbf{p}_a^H \mathbf{\hat{H}}_2 \mathbf{p}_a + \sigma_n^2}{\sigma_n^2} \) is monotonically increasing with \( \mathbf{p}_a \), the constraint (9b) must be active to satisfy the optimal solution, i.e., \( \mathbf{p}_a^H \mathbf{\hat{H}}_2 \mathbf{p}_a = \mathbf{p}_a^{max} \).

Therefore, the problem (9a) can be transformed to a generalized Rayleigh problem as follows

\[
\begin{align*}
\text{maximize} & \quad \frac{\mathbf{p}_a^H \mathbf{\hat{H}}_1 \mathbf{p}_a}{\mathbf{p}_a^H \mathbf{\hat{H}}_3 \mathbf{p}_a^{max}} \\
\text{subject to} & \quad \mathbf{p}_a^H (\mathbf{\hat{H}}_3 / \mathbf{p}_a^{max}) \mathbf{p}_a = 1.
\end{align*}
\]

Moreover, due to the normalized constraint (14b), we can reformulate the problem (14) as an unconstrained equivalent problem as follows

\[
\begin{align*}
\text{maximize} & \quad \frac{\mathbf{p}_a^H \mathbf{\hat{H}}_1 \mathbf{p}_a}{\mathbf{p}_a^H (\mathbf{\hat{H}}_3 / \mathbf{p}_a^{max}) \mathbf{p}_a} \\
\text{subject to} & \quad \frac{\mathbf{p}_a^H (\mathbf{\hat{H}}_3 / \mathbf{p}_a^{max}) \mathbf{p}_a}{\mathbf{p}_a^H \mathbf{\hat{H}}_1 \mathbf{p}_a} = \lambda.
\end{align*}
\]

Denoting \( \lambda \) as a feasible objective value of (15), we have

\[
\frac{\mathbf{p}_a^H \mathbf{\hat{H}}_3 \mathbf{p}_a}{\mathbf{p}_a^H (\mathbf{\hat{H}}_3 / \mathbf{p}_a^{max}) \mathbf{p}_a} = \lambda.
\]

Then, the relationship of \( \mathbf{\hat{H}}_1 \mathbf{p}_a = \lambda (\mathbf{\hat{H}}_3 / \mathbf{p}_a^{max}) \mathbf{p}_a \) shows that optimal \( \mathbf{p}_a \) can be solved via the GEVD of \( \mathbf{\hat{H}}_1 \) and \( \mathbf{\hat{H}}_3 / \mathbf{p}_a^{max} \) to obtain the largest eigenvalue \( \lambda^{max} \) and the corresponding eigenvector as solution.

### IV. ANALYSIS OF CONVERGENCE AND COMPLEXITY

#### A. Analysis of Convergence

We focus on the convergence of the proposed QT-based iterative algorithm because the convergence of FPI has been proved in [11]. Since we obtain the optimal \( \mathbf{p}_a \) and \( \mathbf{y} \) in each iteration, monotonic convergence of the proposed algorithm is guaranteed. Furthermore, the sum rate is upper bounded owing to the finite total power. Therefore, the proposed QT-based algorithm is guaranteed to converge to at least a locally optimal solution of (9a). Overall, the proposed
two-step optimization with FPI-based and QT-based methods guarantees the convergence.

B. Analysis of Computational Complexity

In the optimization of phase \( \varphi \), FPI and SDR have the complexity of \( \mathcal{O}(I_{itr}^{FPI} M^2) \) and \( \mathcal{O}(M^{1.5}) \), respectively. In addition, we also compare to the other algorithms that can iteratively solve the problem (7a). The gradient descent (GD) method [14] using the gradient to update on the phase directly has the complexity of \( \mathcal{O}(I_{itr}^{GD} M^2) \). The ADMM method [13] using separated variables in the augmented lagrangian function has the complexity of \( \mathcal{O}(I_{itr}^{ADMM} M^3) \). Note that \( I_{itr}^{(2)} \) is the number of iterations to satisfy the stopping criteria in different algorithms. Although GD has the same complexity order as FPI, it incurs larger iterations to converge. Therefore, FPI has the lowest complexity, which will be shown in numerical results. In the optimization of amplification \( p_a \), QT and GEVD have the complexity of \( \mathcal{O}(I_{itr}^{QT} M^3) \) and \( \mathcal{O}(I_{itr}^{GEVD} M^2 + M^3) \) in QT and GEVD algorithms.

V. Simulation Results

In this section, we present the numerical results of the two-step optimization to show the efficiency in the active-IRS NOMA system. Unless specified otherwise, the system setups are referring to [3] and setting as follows. The IRS comprises \( M = 16 \) to serve \( K = 3 \) users. As shown in Fig.1(b), in 2-dimensional xy-plane, the BS, IRS are located at \((0,0)\), \((50,0)\) in meter (m), respectively. The \( K \) users are uniformly and randomly distributed within a semicircle with radius of 25 m centered at \((50,0)\) m. The BS-IRS link is modeled by Rician fading channel with Rician factor as 5 because the IRS can be deployed in a location with strong LoS in advance. On the other hand, the IRS-UE links adopt the Rayleigh fading channel whose random elements are randomly generated from the independent and identical distribution of \( \mathcal{CN}(0,1) \). The large-scale path loss can be modeled as \( \beta(d) = \beta_0(d/d_0)^{-\alpha} \), where \( \beta_0 \) is the path loss at the reference distance \( d_0 \), and \( \alpha \) denotes the path loss exponent. We set \( d_0 = 1 \) and \( \beta_0 = -30 \) dB, and the path loss exponent of the BS-IRS link and IRS-UE link are 2.4, 2.8, respectively. Let the transmit power of each user \( p_{k}^{max} = 20 \) dBm and total transmit power of active IRS \( p_a^{max} = -10 \) dBm. We set the noise variance \( \sigma_n^2 = \sigma_z^2 = -114 \) dBm. All results are averaged over 5000 independent channel realizations.

In the beginning, we demonstrate the efficient FPI method in the phase optimization. Fig. 2 shows the efficiency of NOMA system in passive-IRS and compares with OMA scheme whose sum rate can be achieved by

\[
R_{sum}^{OMA} = \frac{1}{K} \sum_{k=1}^{K} \log_2 \left( 1 + \frac{K|g^H \Theta_k h_k|^2 p_k}{\sigma_n^2} \right),
\]

where \( \Theta_k = \text{diag}(e^{j\varphi_k}) \) is the optimal IRS configuration to serve the only \( k \)-th user in the separate time slot. It can be observed that NOMA system outperforms OMA system with the increased \( K \) because the passive-IRS serves all users in the same resource block simultaneously. Moreover, FPI can achieve as high performance as SDR but with extreme low-complexity. To verify the low-complexity of FPI, the averaged CPU time is summarized in Table I, where all algorithms are executed on a 3.60GHz Intel Core i7 PC with 32GB RAM.

Then, we show the advantage of the active-IRS in different aspects in Fig. 3. We evaluate the sum rate over different number of reflecting elements in Fig. 3(a) with some observations. Note that we also include the equal power allocation strategy as the baseline. First, active-IRS superior to passive-IRS and improves the performance efficiently with additional low power \( p_a^{max} \) owing to the signal amplification. Although the performance of active-IRS saturates in the high \( M \) region due to the less power allocated in each element, the efficiency in the low \( M \) region is more suitable in practical applications thanks to the lower hardware cost and pilot overhead in channel estimation. Secondly, we can observe that the low-complexity GEVD has negligible performance degradation compared with the QT in most scenarios, which shows the robustness of the upper bound optimization in GEVD. Furthermore, for a fair comparison, we consider the energy efficiency (EE) metric which is defined as [7]

\[
\eta = \frac{R_{sum}^{OMA}}{\frac{1}{2} \sum_{k=1}^{K} p_k^{max} + K P_{UE} + P_{BS} + M P_{IRS} + \frac{1}{4} p_a^{max}}. \tag{17}
\]

The parameters of the power consumption model are summarized in Table II [15]. Fig. 3(b) shows that the EE of passive-IRS saturates or even becomes worse in high \( M \) region. The reason is that the additional reflecting element with linear extra circuit power \( P_{IRS} \) cannot provide linear gain in sum rate. The EE result also indicates that low \( M \) is more efficient for active-IRS. Moreover, Fig. 3(c) shows the deployment of the IRS. Passive-IRS prefers to be placed close to either BS or UE to alleviate the double path loss. On the other hand, active-IRS prefers to be placed far from the transmitter, i.e., UE, since the weaker signals arrived at the IRS planar can achieve larger amplification. However, the upper bound approximation become loose when active-IRS is close to BS because the larger \( p_a \) leads to the unsatisfaction of \( p_a^H H_2 p_a \ll \sigma_n^2 \).

<table>
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<tr>
<th>Algorithm</th>
<th>FPI</th>
<th>SDR</th>
<th>ADMM</th>
<th>GD</th>
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<td>859.3</td>
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<td>100%</td>
<td>0.36%</td>
<td>0.86%</td>
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</tbody>
</table>

Fig. 2. Sum rate of passive-IRS OMA/NOMA versus number of users.

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In summary, with the aid of active-IRS, the system only need to supply small portion of extra power consumption to compensate the double fading effect. Therefore, active-IRS shows potential to achieve green communication efficiently.

VI. CONCLUSION

In this letter, to improve the sum rate, we exploit the advantages of the signal amplification of active-IRS and the resource block sharing of the NOMA scheme simultaneously in the uplink system. We propose a low-complexity two-step optimization to tackle the non-convex sum rate maximization problem. The phase and amplification problems are solved separately in efficient ways. In summary, the two-step optimization for active-IRS achieves higher performance than passive-IRS in the uplink NOMA system, and the friendly hardware overhead is suitable for practical implementation. Moreover, the proposed QT and GEVD methods have the advantage of convergence and low-complexity, respectively, to be applied in different scenarios, suitable for future IoT applications.

APPENDIX PROOF OF LEMMA 1

Proof: Suppose that \( \{ P_\alpha^*, \Theta^*, \{ p_k^* \} \} \) is the optimal solution for problem (4) with \( p_k^* < p_{k,\text{max}}^m, \forall k \in K \). Then, we can always construct a new solution \( \{ \tilde{P}_\alpha, \tilde{\Theta}, \{ \tilde{p}_k \} \} \) which satisfies \( \tilde{\Theta} = \Theta^* \), \( \tilde{p}_k = c^2 p_k^* \leq p_{k,\text{max}}^m \) and \( \tilde{P}_\alpha = P_\alpha^*/c \) with \( c > 1 \). It can be verified that the constraint (4c) is still satisfied as

\[
\sum_{k=1}^{K} \| \tilde{P}_\alpha \tilde{\Theta} h_k \sqrt{\tilde{p}_k} \|^2 + \sigma_n^2 \| \tilde{P}_\alpha \tilde{\Theta} \|^2 < \sum_{k=1}^{K} \| P_\alpha^* \Theta^* h_k \sqrt{p_k^*} \|^2 + \sigma_n^2 \| P_\alpha^* \Theta^* \|^2 \leq p_{\text{max}}^m.
\]

Furthermore, we obtain the objective relationship as

\[
\sum_{k=1}^{K} \| g^H P_\alpha^* \Theta^* h_k \sqrt{p_k^*} \|^2 / \sigma_n^2 < \sum_{k=1}^{K} \| g^H \tilde{P}_\alpha \tilde{\Theta} h_k \sqrt{\tilde{p}_k} \|^2 / \sigma_n^2 \leq \| g^H \tilde{P}_\alpha \tilde{\Theta} \|^2 / \sigma_n^2 \leq \| g^H P_\alpha^* \Theta^* \|^2 / \sigma_n^2.
\]

REFERENCES