Structured Random Compressed Channel Sensing for Millimeter-Wave Large-Scale Antenna Systems

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Abstract—In millimeter-wave (mmWave) communications, large-scale antenna system (LSAS) is considered an essential technology to realize beamforming gain to compensate for huge propagation loss. However, channel estimation for LSAs poses a formidable challenge, especially when hybrid analog–digital structures are adopted for ensuring reasonable complexity and cost. To address this challenge, compressed channel sensing (CCS) is leveraged to measure mmWave channels via a random sensing codebook. By exploiting the sparse nature in mmWave channels, only a small number of measurements is required for channel recovery. However, signaling or storing the configuration of a full random sensing codebook leads to a huge burden. Moreover, because the sensing beam of a full random sensing codebook always spreads its power over the channel, the CCS has a stringent requirement on the signal-to-noise ratio (SNR) for robust channel recovery. To overcome these issues, we propose a structured random sensing codebook inspired by the random convolutional measurement process. Owing to its structured nature, signaling or storing overhead from the codebook configuration is significantly reduced. Additionally, the structured random sensing codebook can concentrate its power in a local angle coverage for a sectorized cell, thus improving the robustness in low-SNR regimes. Simulation results demonstrate that the recovery performance of the proposed structured random sensing codebook design is comparable to that of the full random design. Regarding the robustness in low-SNR regimes, the recovery performance is substantially improved by the structured random sensing codebook with power concentration for a local angle coverage.

Index Terms—Millimeter-wave communications, large-scale antenna systems, channel estimation, compressed channel sensing, structured random compressed sensing.

I. INTRODUCTION

T he rapid growth of mobile traffic has imposed tremendous demands of high data rates on fifth-generation (5G) cellular systems. Toward this end, millimeter-wave (mmWave) communications are the key enabling techniques that utilize the abundant spectrum resource [1]. In mmWave communications, large-scale antenna system (LSAS) is fundamental at both base stations (BSs) and user equipment (UE) to provide beamforming gain to compensate for the huge propagation loss at mmWave bands [2], [3]. For ease of implementation, hybrid analog–digital (AD) LSAS [4]–[6] driven by a small number of radio frequency (RF) chains is a feasible solution to avoid the fabrication cost and energy consumption of a large number of high-frequency mixed-signal components used in all antennas. Moreover, the hybrid AD LSAS is capable of realizing both beamforming and spatial multiplexing gains via the array signal processing in both analog and digital domains.

To exploit the full benefits of the LSAS, channel knowledge is crucial for both BSs and UE. However, the large number of antennas in the LSAS leads to a large size of the channel matrix, which results in huge resource burdens for channel estimation. Moreover, traditional reference signal (RS)-based training is not applicable to the hybrid AD LSAS because the baseband cannot directly access the individual signal corresponding to each antenna. By exploiting the fact that mmWave channels tend to have few dominant propagation paths owing to the limited scattering effect [7], a closed-loop beam training referred to as hierarchical channel sensing (HCS) [8]–[10] is proposed to address the challenge of channel estimation in the LSAS. In HCS, a divide-and-conquer search process is carried out across the hierarchy of multi-resolution beams, and the desirable beams are refined persistently according to the UE report until reach the target resolution. HCS can greatly reduce the sweeping overhead (i.e., number of employed resources for beam sweeping) when the number of levels in the hierarchy is unlimited. However, considering control complexity and feedback overhead, the number of levels is limited in practice, thus the overhead reduction of is minor. Moreover, the sweeping overhead of HCS linearly increases with the number of UEs since the BS needs to sweep every possible beam according to the report from each UE.

Recently, an open-loop beam training called compressed channel sensing (CCS) has been explored in [11]–[20] by leveraging the tools of compressed sensing [21]–[23]. In CCS, the random sensing codebook is swept to measure the channel. Each sensing beam (i.e., random beamforming vector) of the sensing codebook is a set of unimodular entries with quantized random phases, and thus it can be realized in analog phase-shifting arrays of the hybrid AD LSAS. By exploiting the sparse nature in mmWave channels, CCS only requires a small number of measurements for channel recovery. As CCS works in an open-loop...
manner, it does not need complicated control protocol and UE report within beam sweeping, and enables simultaneous training for multiple UE without increasing the sweeping overhead. Recently, CCS has been further extended to frequency-selective wideband channel estimation [15], [16], and channel covariance estimation [17], [18] for the LSAS. CCS for estimating the mmWave channels with structured sparsity has also been investigated in [19], [20]. However, two critical issues in CCS need to be addressed in practice:

1) **Signaling/storage overhead of codebook configuration:**
In CCS, the knowledge of both transmit and receive sensing codebooks is necessary for channel recovery. Thus, the configuration of the sensing codebook adopted at the BS should be known to the UE. However, the configuration of the full random sensing codebook must be explicitly signaled to the UE or stored in the UE owing to its completely unstructured nature. It leads to an unbearable signaling or storage overhead. In many works, e.g., [24]–[27], this issue has also been indicated, and some sensing matrices with deterministic constructions have been proposed to avoid it. However, these solutions usually require additional overhead (e.g., hardware and latency) to generate the sensing matrices on-the-fly. Moreover, most of them cannot guarantee comparable recovery performance to that of the full random sensing matrix [28].

2) **Stringent requirement on SNR for robust channel recovery:**
The sectorized cellular systems are typically adopted for outdoor deployments, especially when LSASs are considered [29]–[31]. In a sectorized cell, the BS adopts a LSAS to serve a local angle coverage instead of a full angle coverage to increase directivity gain, enhance capacity by spatial division [49], and avoid the limitation of beam scanning coverage [32]–[34]. However, as shown in Fig. 1(a), the sensing beam of the full random sensing codebook always spreads its power over a full angle coverage. Without directivity gains, CCS has a stringent requirement on signal-to-noise ratio (SNR) since channel recovery has the minimum requirement on the ratio of signal power after measurement to noise power [35]. It severely limits the operating regime of CCS, especially for mmWave with significant propagation loss.

In this paper, we aim to leverage the benefits of CCS for channel estimation in mmWave LSASs. To address the two aforementioned issues, we present a new class of CCS inspired by the structured random compressed sensing [36]–[38]. The main contributions of this paper are summarized as follows:

- We propose a novel dual-stage sensing codebook consisting of a set of sampling beams randomly selected from the discrete Fourier transform (DFT) codebook for random subsampling, and a modulation with a spreading sequence in the spatial domain for random convolution. Due to the structured nature, configuration of the proposed sensing codebook can be implicitly represented via (i) the sampling indices of the selected DFT beams, and (ii) the spreading sequence, rather than explicit matrix. Thus, the overhead from signaling or storing the proposed sensing codebook can be reduced from $O(MN)$ to $O(M \log_2 N)$, where $M$ and $N$ denote the numbers of measurements and antennas, respectively.
- We present a random convolutional measurement process extended from one class of compressed sensing tools [36]–[38] for the CCS adopting the proposed sensing codebooks. Then, a theoretical bound derived from this measurement process is offered to show that the proposed sensing codebook design compares favorably to the full random design.
- We further design a sensing codebook with the capability of power concentration based on the proposed dual-stage codebook structure. In the proposed sensing codebook design, we can configure the direction and the beamwidth of each sensing beam by simply adjusting the direction of its sampling beam and the spreading factor of its spreading sequence, respectively. As illustrated in Fig. 1(b), given a local angle coverage for a sectorized cell, we can design a structured random sensing codebook to concentrate its power in the intended angle coverage to increase directivity gains. Thus, the robustness for channel recovery can be improved in low-SNR regimes.
- We also show that the CCS adopting the proposed sensing codebook with power concentration in a local angle coverage can also achieve stable recovery, and offer the corresponding theoretical bound. Meanwhile, based on the theoretical bound, the design of spreading factor for a local angle coverage is derived.

Finally, we present simulation results on the recovery performance of the proposed strategy and show that it can alleviate the stringent requirement on SNR for robust channel recovery.

The rest of this paper is organized as follows. In Section II, the system and channel models are introduced. The structured random sensing codebook design is presented in Section III. In Section IV, the structured random sensing codebook with
power concentration for a local angle coverage is introduced. Simulation results are presented in Section V. Finally, the conclusions are drawn in Section VI.

II. CHANNEL AND SYSTEM MODELS

A. Notation
Throughout this paper, a normal-faced letter $a$ denotes a scalar, a bold-faced lowercase letter $a$ denotes a vector, and a bold-faced uppercase letter $A$ denotes a matrix. Other operations used in this paper are defined as follows:
- $A_{p:q,r:q}$ is the $p$-th row vector, $r$-th column vector, and $(p,q)$-th entry of $A$, respectively.
- $A^\Omega$ is the submatrix formed by collecting the columns with indices in the set $\Omega$.
- $A^{\Omega,\Omega}$ is the submatrix formed by taking a block of the entries of $A$ whose row and column are indexed by $\Omega$.
- $A^T$, $A^*$, $A^H$, $A^{-1}$, and $A^1$, and vec$(A)$ denote the transpose, conjugate, conjugate-transpose, inverse, pseudo-inverse, and vectorization of $A$, respectively.
- $\otimes$ is the Kronecker product of $A$ and $B$.
- $E[|\cdot|]$, $|\cdot|$, $||\cdot||_1$, $||\cdot||_2$, and $||\cdot||_F$ denote the expectation operation, element-wise absolute value or cardinality for a set, 1-norm, 2-norm, and Frobenius norm, respectively.
- $F_N$ denotes a unitary DFT matrix with size of $N$, where $F_N^H = (N^*)^{-1/2}e^{-2\pi j(p-1)(q-1)/N}$.
- $C = Circ(c)$ denotes a circulant matrix specified by the sequence $c$, which appears as the first column of $C$.

B. Physical and Virtual Angular Domain Channel Models

In an open-loop beam training, each UE can be considered an independent subsystem. Without loss of generality, we consider the LSAS with a BS with $N_T$ antennas and a reference UE with $N_R$ antennas. Both are equipped with half-wavelength-spaced uniform linear arrays (ULAs).

In this study, we use the cluster-based physical channel model, which is also widely used in the literatures [11]–[18], to capture the limited scattering channel. In this model, the physical channel is assumed to be the sum of $K$ clusters. Since the measurement campaigns in dense-urban environments [7] have revealed that mmWave channels typically have only a few scattering clusters, and the angle spreading within each cluster is also small. Thus, we can assume that $K \ll N_R \leq N_T$, and each cluster contributes a single dominant propagation path between the BS and the UE because most power of each cluster is concentrated around boresight direction. Under these assumptions, a narrowband block-fading channel can be expressed as

$$H = \sqrt{\frac{N_TN_R}{K}} \sum_{k=1}^{K} \alpha_k a(N_R, \theta_k^{AoA}) a^H(N_T, \theta_k^{AoS}),$$

where $H \in \mathbb{C}^{N_R \times N_T}$ is the physical channel between the BS and the UE, $\alpha_k$ is the complex gain of the $k$-th propagation path. The steering vectors $a(N_R, \theta_k^{AoA}) \in \mathbb{C}^{N_R \times 1}$ and $a(N_T, \theta_k^{AoS}) \in \mathbb{C}^{N_T \times 1}$ correspond to the physical angle-of-arrival (AoA) $\theta_k^{AoA}$ and the angle-of-departure (AoD) $\theta_k^{AoS}$ of the $k$-th propagation path for the ULAs with $N_T$ and $N_R$ antennas, respectively. Both physical AoDs and AoAs are considered continuous and uniformly distributed within the intended angle coverages, respectively. The complex gains the complex gains $\{\alpha_k\}_{k=1}^{K}$ are assumed to be i.i.d. $(0, \sigma^2_{\alpha})$ for $k = 1, \ldots, K$, where $\sigma^2_{\alpha}$ is the average power of the $k$-th cluster. To constrain the total channel power, i.e., $\mathbb{E}||H||_F^2 = N_T N_R$ [39], [40], the sum of average cluster power should satisfy $\sum_{k=1}^{K} \sigma^2_{\alpha} = K$.

To apply compressed sensing for channel estimation, we use the virtual angular domain (VAD) representation [26] to provide a discrete approximation to the physical channel (1). Instead of taking the AoDs and AoAs from arbitrary angles, the VAD representation quantizes them via fixed virtual angle grids with finite resolutions. Using the VAD representation, we can approximate the physical channel (1) by

$$\tilde{H} \approx A R \tilde{H}_v A_T^H,$$

where $\tilde{H}_v \in \mathbb{C}^{G_R \times G_T}$ is the VAD channel matrix, $A_T \in \mathbb{C}^{N_T \times G_T}$ and $A_R \in \mathbb{C}^{N_R \times G_R}$ denote the VAD transformation dictionaries consisting of the steering vectors corresponding to the transmit and receive virtual angle grids with resolutions $G_T$ and $G_R$, respectively. To effectively approximate the physical channel (1), both transmit and receive virtual angle grids should be applied with sufficient high resolutions, such that $G_T \geq N_T$ and $G_R \geq N_R$. When $G_T = N_T$ and $G_R = N_R$, $A_T$ and $A_R$ can be represented as the unitary DFT matrices $F_{N_T}$ and $F_{N_R}$.

The VAD representation (2) is defined for a channel with full transmit and receive angle coverages. However, in practical cellular systems, because outdoor deployment typically adopts the sectorized cellular systems [29]–[31], the constraint on AoD should be considered during channel estimation. Based on the prior knowledge of local transmit angle coverage corresponding to a sectorized cell, we can directly consider that the transmit virtual angle grid out of the local transmit angle coverage have no energy. Thus, given a local transmit angle coverage, we have a subsequence $\Theta_T \subset \{1, \ldots, G_T\}$ such that

$$\tilde{H}_v^{\theta} = 0, \text{ if } q \notin \Theta_T.$$  

In contrast, the AoA should be considered within a full angle coverage because the location and orientation of each UE are random. Finally, let $H_{\Theta_T} = \tilde{H}_v^{\Theta_T} \in \mathbb{C}^{G_R \times \Theta_T}$, we can rewrite (2) as

$$\tilde{H} = A_R H_{\Theta_T} (A_T^{\Theta_T})^H.$$  

C. Channel Estimation by Compressed Channel Sensing

In this work, we aim to study the sensing codebook design for CCS. As the sensing beams of the sensing codebook are typically formed in the analog domain, they can be directly adopted for both narrowband CCS [11]–[14] and wideband CCS [15]–[18]. To focus on the sensing codebook design and make the paper concise, we consider only the narrowband CCS, and the extensions to wideband CCS are straightforward.

In the $m$-th measurement of the CCS, the BS forms a unit-norm transmit sensing beam $p_m \in \mathbb{C}^{N_T \times 1}$, and the UE forms a unit-norm receive sensing beam $q_m \in \mathbb{C}^{N_R \times 1}$ to measure the
channel. Considering the physical channel (1), we have
\[ y_m = \sqrt{\rho} q_m^H h_m p_m s_m + q_m^H n_m \in \mathbb{C}^{1 \times 1} \]
\[ (a) = \sqrt{\rho} (p_m^T \otimes q_m^H) \text{vec}(H) s_m + q_m^H n_m, \]
where (a) follows from the vectorization originally introduced in [8], \( s_m \in \mathbb{C}^{1 \times 1} \) is the \( m \)-th RS transmitted through the \( m \)-th transmit sensing beam, \( n_m \in \mathbb{C}^{N_h \times 1} \) is a Gaussian noise vector with variance \( \sigma_n^2 \) in the measurement, and \( \rho \) denotes the average received power before measurement. Thus, the SNR for each measurement is represented as \( \rho/\sigma_n^2 \).

During the beam sweeping for CCS, the sensing codebooks \( \mathbf{P} = [p_1, \ldots, p_M] \in \mathbb{C}^{N_T \times M} \) and \( \mathbf{Q} = [q_1, \ldots, q_M] \in \mathbb{C}^{N_h \times M} \) with \( M \) transmit and receive sensing beams are swept at the BS and the UE, respectively. Since CCS adopts sensing beams instead of orthogonal RSs to measure the channel, the transmitted RSs are typically identical within the beam sweeping, i.e., \( s_m = 1 \) for \( m = 1, \ldots, M \). After all the sensing beams are swept, we have a measurement vector \( y = [y_1, \ldots, y_M]^T \in \mathbb{C}^{M \times 1} \) by stacking the \( M \) measurements
\[ y = \sqrt{\rho} \left[ \begin{array}{c} p_1^T \otimes q_1^H \\ \vdots \\ p_M^T \otimes q_M^H \end{array} \right] \text{vec}(H) + \left[ \begin{array}{c} q_1^H n_1 \\ \vdots \\ q_M^H n_M \end{array} \right]. \] (7)

To formulate a channel recovery problem from (7), we apply the VAD representation (2) into (7) [13]. Then, we have
\[ y \approx \sqrt{\rho} \left[ \begin{array}{c} p_1^T \otimes q_1^H \\ \vdots \\ p_M^T \otimes q_M^H \end{array} \right] (A_T \otimes A_R) \text{vec}(\mathbf{H}_w) + \left[ \begin{array}{c} q_1^H n_1 \\ \vdots \\ q_M^H n_M \end{array} \right] \]
\[ \approx \sqrt{\rho} \Phi \mathbf{H}_w + \mathbf{e}. \] (8)

Herein, \( \Phi \in \mathbb{C}^{M \times N} \) denotes the equivalent measurement matrix, \( \mathbf{e} \in \mathbb{C}^{N \times 1} \) is the noise vector, and \( \mathbf{H}_w = \text{vec}(\mathbf{H}_w) \in \mathbb{C}^{N \times 1} \) is a VAD channel vector, where \( N = G_T G_R \). Since we have \( M < N \) in the CCS, (8) is underdetermined. However, when \( \mathbf{H}_w \) exhibits sparsity/compressibility, it can be recovered by the \( \ell_1 \) optimization
\[ \mathbf{H}_w = \arg \min_{\mathbf{H}_w} \| \mathbf{H}_w \|_1, \text{ s.t. } \| y - \sqrt{\rho} \Phi \mathbf{H}_w \|_2^2 \leq \varepsilon. \] (9)

The variable \( \varepsilon \) bounds the noise effect in the measurements of the CCS. Several greedy-based recovery algorithms [41]–[43] have been proposed to find approximated solutions of the \( \ell_1 \) optimization in low complexity. According to the compressed sensing theory [21]–[23], stable channel recovery is guaranteed if the equivalent measurement matrix \( \Phi \) satisfies the restricted isometry property (RIP).

**Definition 1 (\( (K, \delta) \text{– RIP} \ [21]) \):** An \( M \times N \) measurement matrix \( \Phi \) satisfies the \( (K, \delta) \)–RIP if
\[ (1 - \delta) \| \mathbf{H}_w \|_2^2 \leq \| \Phi \mathbf{H}_w \|_2^2 \leq (1 + \delta) \| \mathbf{H}_w \|_2^2, \]
for all \( K \)-sparse \( \mathbf{H}_w \).

The RIP ensures that all submatrices of \( \Phi \) with size \( M \times K \) are close to an isometry, and are therefore, distance-preserving. Moreover, the RIP leads to stability with respect to the non-ideal measurements introduced by the additive noise and the measurement matrix mismatch. Once \( \mathbf{H}_w \) is derived from channel recovery, the estimated channel matrix can be given by
\[ \hat{\mathbf{H}} = A_R \text{vec}^{-1}(\mathbf{h}_w) A_T^H. \] (11)

In the CCS, it is worth mentioning that the VAD channel vector \( \mathbf{h}_w \) typically exhibits compressibility rather than purely sparsity because the physical AoAs \( \{\theta_k^{AoA}\} \) and AoDs \( \{\theta_k^{AOD}\} \) are actually continuous. Fortunately, as proved in [30], the RIP restriction guarantees stable recovery whether the signal (or channel) is sparse or compressible.

### Table I: Feature Comparison of Different Sensing Codebook Designs

<table>
<thead>
<tr>
<th>Features</th>
<th>Full random Sensing codebook</th>
<th>DFT-MTC Sensing codebook [45]</th>
<th>Proposed Sensing codebook</th>
</tr>
</thead>
<tbody>
<tr>
<td>Theoretical recovery guarantee</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>No. of required measurements for stable recovery</td>
<td>( O(\delta^2 K \log N) )</td>
<td>( NA )</td>
<td>( O(\delta^2 K \log K \log N) )</td>
</tr>
<tr>
<td>Sweeping overhead</td>
<td>( O(M) )</td>
<td>( O(N(N_n \log N)) )</td>
<td>( O(M) )</td>
</tr>
<tr>
<td>Signaling (or storage) approach and overhead</td>
<td>Explicit</td>
<td>Implicit</td>
<td>Implicit</td>
</tr>
<tr>
<td>Beamwidth of the sensing beam</td>
<td>Full angle coverage</td>
<td>Narrow angle coverage</td>
<td>Configurable</td>
</tr>
</tbody>
</table>
size of the DFT codebook, and it has no theoretical guarantee for stable recovery when only a few measurements are given.

To address the two issues, a new class of CCS inspired by the structured random compressed sensing [36]–[38] is presented in this paper. To clarify the differences between prior work and the proposed approach, we briefly summarize the comparisons in Table I.

III. STRUCTURED RANDOM SENSING CODEBOOK BASED ON THE RANDOM CONVOLUTIONAL MEASUREMENT PROCESS

A. Proposed Structured Random Sensing Codebook Design

In the proposed sensing codebook, the transmit sensing beam \( p_m \) and receive sensing beam \( q_m \) for the \( m \)-th measurement are constructed by the sampling beams \( \{ w_{T,m} \in \mathbb{C}^{N_T \times 1}, w_{R,m} \in \mathbb{C}^{N_R \times 1} \} \) and the beam spreaders \( \{ \Delta_T \in \mathbb{C}^{N_T \times N_T}, \Delta_R \in \mathbb{C}^{N_R \times N_R} \} \), respectively. Then, the sensing beams can be expressed as

\[
\{ p_m, q_m \} = \{ \Delta_T w_{T,m}, \Delta_R w_{R,m} \}.
\]

We now delve into the details of dual-stage codebook structure:

- **Sampling beamformer:** In this stage, \( w_{T,m} \) and \( w_{R,m} \) are selected from the transmit and receive DFT codebooks through the sampling indices \( \Omega_{T,m} \) and \( \Omega_{R,m} \), as illustrated by the corresponding beam patterns (blue) in Fig. 2, respectively. The sampling beams can be expressed as

\[
\{ w_{T,m}, w_{R,m} \} = \begin{cases} F_{NT,m} \Omega_{T,m}, & F_{NR,m} \Omega_{R,m} \end{cases},
\]

where \( r_{T,m} \in \mathbb{R}^{1 \times N_T} \) and \( r_{R,m} \in \mathbb{R}^{1 \times N_R} \) denote the sampling vectors for selecting the columns of \( F_{NT} \) and \( F_{NR} \) indexed by \( \Omega_{T,m} \) and \( \Omega_{R,m} \), respectively.

- **Beam spreader:** In this stage, \( \Delta_T \) and \( \Delta_R \) represent the modulations in the spatial domain with the spreading sequences \( \sigma_T \in \mathbb{C}^{N_T \times 1} \) and \( \sigma_R \in \mathbb{C}^{N_R \times 1} \), respectively. Then, they can be expressed as

\[
\{ \Delta_T, \Delta_R \} = (\text{diag}(\sigma_T), \text{diag}(\sigma_R)) .
\]

To address the two issues, a new class of CCS inspired by the structured random compressed sensing [36]–[38] is presented in this paper. To clarify the differences between prior work and the proposed approach, we briefly summarize the comparisons in Table I.

**Algorithm 1: Structured Random Sensing Codebook Design.**

**Input:** \( N_T, N_R, \) and \( M \)

1: Generate a subset \( \Omega_G \) with \( M \) different indices downselected from \( \{1, \ldots, N_T N_R\} \) uniformly at random.

**return** subsampling index set \( \Omega_G = [\Omega_{G,1}, \ldots, \Omega_{G,M}] \)

**Base Station:**

**Input:** \( N_T \) and \( \Omega_G \)

1: Compute \( \Omega_{T,m} = [(\Omega_{G,m} - 1)/N_R] + 1, \) for \( m = 1, \ldots, M \)

2: Generate a spreading sequence \( \sigma_T \) with the length of \( N_T \)

3: Compute \( \Delta_T = \text{diag}(\sigma_T) \)

4: Compute \( P = \Delta_T F_{NT} R_{\Omega_T} \)

**return** transmit sensing codebook \( P = [p_1, \ldots, p_M] \)

**User Equipment:**

**Input:** \( N_R \) and \( \Omega_G \)

1: Compute \( \Omega_{R,m} = [(\Omega_{G,m} - 1)\text{mod} N_R] + 1, \) for \( m = 1, \ldots, M \)

2: Generate a spreading sequence \( \sigma_R \) with the length of \( N_R \)

3: Compute \( \Delta_R = \text{diag}(\sigma_R) \)

4: Compute \( Q = \Delta_R F_{NR} R_{\Omega_R} \)

**return** receive sensing codebook \( Q = [q_1, \ldots, q_M] \)

In the proposed design, the spreading sequence is should be a unimodular sequence with good autocorrelation property, such as the maximum-length (ML) sequence or the Zadoff–Chu (ZC) sequence. After the beam spreader, the sampling beams \( w_{T,m} \) and \( w_{R,m} \) are spread over full angle coverages, as illustrated by the corresponding beam patterns (blue) in Fig. 2.

To generate a structured random sensing codebook consisting of \( M \) sensing beams, these sensing beams are constructed by \( M \) sampling beams and a same beam spreader based on (12). Thus, we can express the transmit and receive sensing codebooks as

\[
\{ P, Q \} = \{ \Delta_T F_{NT} R_{\Omega_T}, \Delta_R F_{NR} R_{\Omega_R} \} ,
\]

respectively. The matrices \( R_{\Omega_T} = [r_{T,1}^T, \ldots, r_{T,M}^T] \in \mathbb{R}^{M \times N_T} \) and \( R_{\Omega_R} = [r_{R,1}^T, \ldots, r_{R,M}^T] \in \mathbb{R}^{M \times N_R} \) denote the sampling operators for selecting \( M \) sampling beams.
from $\mathbf{F}_{N_T}$ and $\mathbf{F}_{N_R}$ according to the sampling index sets $\Omega_T = \{\Omega_{T,1}, \ldots, \Omega_{T,M}\}$ and $\Omega_R = \{\Omega_{R,1}, \ldots, \Omega_{R,M}\}$, respectively. Similarly to (7), we can stack the $M$ measurements in a vector form after the sweeping of $\mathbf{P}$ and $\mathbf{Q}$ in (15)

$$
\mathbf{y} = \sqrt{\rho} \left[ (\Delta_T \mathbf{F}_{N_T} \mathbf{r}_{T,1}^{T}) \otimes (\Delta_R \mathbf{F}_{N_R} \mathbf{r}_{R,1}^H) ight] \mathbf{v}_e(\mathbf{H}) + \mathbf{e}
$$

$$
= \sqrt{\rho} \left[ \begin{array}{c}
\mathbf{r}_{T,1} \\
\vdots \\
\mathbf{r}_{T,M} \otimes \mathbf{r}_{R,M}
\end{array} \right] \left[ (\Delta_T \mathbf{F}_{N_T})^T \otimes (\Delta_R \mathbf{F}_{N_R})^H \right] \mathbf{v}_e(\mathbf{H}) + \mathbf{e}
$$

$$
= \sqrt{\rho} \left[ \mathbf{r}_{T,1} \cdots \mathbf{r}_{T,M}^T \mathbf{r}_{G,M}^T \right] \left[ (\Delta_T \mathbf{F}_{N_T})^T \otimes (\Delta_R \mathbf{F}_{N_R})^H \right] \mathbf{v}_e(\mathbf{H}) + \mathbf{e}
$$

$$
= \sqrt{\rho} \mathbf{R}_{\Omega_G} \left[ (\Delta_T \mathbf{F}_{N_T})^T \otimes (\Delta_R \mathbf{F}_{N_R})^H \right] \mathbf{v}_e(\mathbf{H}) + \mathbf{e},
$$

(16)

where $\mathbf{R}_{\Omega_G} \in \mathbb{R}^{M \times N_T N_R}$ is the subsampling operator containing the $M$ subsampling vectors $\{\mathbf{r}_{G,m} = \mathbf{r}_{T,m} \otimes \mathbf{r}_{R,m}\}_{m=1, \ldots, M} \in \mathbb{R}^{N_T N_R}$. In the proposed design, $\mathbf{R}_{\Omega_G}$ is given by collecting $M$ rows of $\mathbf{I}_{N_T}$ indexed by $\Omega_G = \{\Omega_{G,1}, \ldots, \Omega_{G,M}\}$, and $\Omega_G$ is downselected from $\{1, \ldots, N_T N_R\}$ uniformly at random. Then, according to $\Omega_G$, the corresponding sampling index sets for sampling operators $\mathbf{R}_{\Omega_T}$ and $\mathbf{R}_{\Omega_R}$ are respectively given by

$$
\{\Omega_{T,m}, \Omega_{R,m}\} = \{(\Omega_{G,m} - 1) / N_R + 1, (\Omega_{G,m} - 1) \bmod N_R + 1\},
$$

(17)

for $m = 1, \ldots, M$. Finally, the design of the structured random sensing codebook for the BS and UE is summarized in Algorithm 1.

B. Practical Implementation in the LSAS

The proposed sensing codebook design has two advantages for practical implementation in the LSAS:

1) Realizable in the analog phase-shifting arrays: To perform the beam sweeping in the analog domain of the hybrid AD LSAS, the sensing beam should meet the constant modulus and quantized phase constraints for realizing in the phase-shifting arrays. In the proposed sensing codebook design, the $m$-th transmit sensing beam of $\mathbf{P}$ in (15) can be rewritten as

$$
\mathbf{p}_m = (N_T)^{-1/2} \left[ e^{-j(\omega_{m,1} + \varphi_1)}, \ldots, e^{-j(\omega_{m,N_T} + \varphi_{N_T})} \right]^T,
$$

(18)

where $\{\omega_{m,\ell}\}_{\ell=1, \ldots, N_T}$ and $\{\varphi_{\ell}\}_{\ell=1, \ldots, N_T}$ are the phase-shifting angles resulting from $\mathbf{w}_{T,m}$ and $\Delta_T$, respectively. From (13), since $\mathbf{w}_{T,m}$ is selected from $\mathbf{F}_{N_T}$ according to $\Omega_{T,m}$, we have $\omega_{m,\ell} = 2\pi(\ell - 1)(\Omega_{T,m} - 1) / N_T$. From (14), if $\sigma_T$ is a bipolar sequence (e.g., ML sequence) in the beam spreader $\Delta_T$, we have

$$
\varphi_{\ell} = \left\{ \begin{array}{ll}
0, & \sigma_{T,\ell} = +1; \\
\pi, & \sigma_{T,\ell} = -1,
\end{array} \right.
$$

(19)

for $\ell = 1, \ldots, N_T$. As a result, the sum of phase-shifting angles $\{\omega_{m,\ell} + \varphi_\ell\}_{\ell=1, \ldots, N_T}$ can be realized in the analog phase-shifting array with $\log_2 N_T$ angle quantization bits. On the other hand, if $\sigma_T$ is a ZC sequence, we have

$$
\varphi_{\ell} = \left\{ \begin{array}{ll}
\pi(\ell - 1)^2 / N_T, & \text{for even } N_T; \\
\pi k(\ell - 1) / N_T, & \text{for odd } N_T,
\end{array} \right.
$$

(20)

for $\ell = 1, \ldots, N_T$. The root index $\tau$ is a positive integer relatively co-prime to $N_T$. In this case, $\{\omega_{m,\ell} + \varphi_\ell\}_{\ell=1, \ldots, N_T}$ can be also realized in the analog phase-shifting array with $1 + \log_2 N_T$ angle quantization bits. As a result, the sensing beams of $\mathbf{P}$ in (15) naturally meet the practical constraints of the hybrid AD hardware. In a similar manner, each sensing beam of $\mathbf{Q}$ in (15) can be also realized in the analog phase-shifting arrays. In the hybrid AD LSAS, each RF chain may be equipped with one analog phase-shifting array. Thus, as illustrated in Fig. 3, multiple sensing beams of the proposed sensing codebooks can be simultaneously swept to reduce the sweeping overhead, and the corresponding RSs are multiplexed by frequency or code divisions.

2) Small signaling/storage overhead owing to the structured nature: In the proposed sensing codebook design, $\mathbf{P}$ can be implicitly represented and reconstructed by (15). In other words, instead of delivering or storing the explicit sensing codebook, we only have to signal or store the sampling index set and the spreading sequence, as shown in Fig. 3. For instance, the configuration of $\mathbf{P}$ comprising $M = 256$ transmit sensing beams for the BS with $N_T = 128$ antennas can be signaled or stored via $\Omega_T$ using $256 \times 7$ bits and $\sigma_T$ using $128$ bits (if $\sigma_T$ is a bipolar sequence), thereby saving 94% signaling or storage burden from that of the Bernoulli random matrix (215 bits should be delivered or stored). Moreover, the spreading sequence can be also represented via an implicit approach. For example, the ZC sequence is generated only according to the root index $\tau$, which can be signaled or stored using $\log_2 N_T$ bits. As a result, compared to the full random
design, the signaling or storage overhead is reduced from $O(MN_T)$ to $O(M \log_2 N_T)$.

C. Equivalent Measurement Process of the Structured Random Compressed Channel Sensing

To explain why stable channel recovery from the CCS approach adopting the structured random sensing codebooks (15) can be achieved, we present an equivalent measurement process for our design. Substituting the VAD representation (2) into (16), we have

$$y \approx \sqrt{\rho R_G} \left( (\Delta_T F_{N_T})^T \otimes (\Delta_R F_{N_R})^H \right) \times \text{vec} (A_R \tilde{H}_\omega A_T^H) + e$$

$$\approx \sqrt{\rho R_G} \left( (\Delta_T F_{N_T})^T \otimes (\Delta_R F_{N_R})^H \right) \times (A_T^* \otimes A_R) \text{vec} (\tilde{H}_\omega) + e$$

$$\approx \sqrt{\rho R_G} \left( (A_T^H \Delta_T F_{N_T})^T \otimes (A_R^H \Delta_R F_{N_R})^H \right) \tilde{H}_\omega + e.$$  \quad (21)

For simplicity, we consider that $G_T = N_T$ and $G_R = N_R$ in this measurement process, and (21) can be rewritten as

$$y \approx \sqrt{\rho R_G} \left( (F_{N_T}^T \Delta_T F_{N_T})^T \otimes (F_{N_R}^T \Delta_R F_{N_R})^H \right) \tilde{H}_\omega + e$$

$$\approx \sqrt{\rho R_G} (C_T^T \otimes C_R^H) \tilde{H}_\omega + e.$$  \quad (22)

From the fact that a circulant matrix can be diagonalized by a unitary DFT matrix, $C_T = \text{Circ}(c_T) \in \mathbb{C}^{N_T \times N_T}$ and $C_R = \text{Circ}(c_R) \in \mathbb{C}^{N_R \times N_R}$ are both circulant matrices, where $c_T = (c_{T,1}, \ldots, c_{T,N_T})^T \in \mathbb{C}^{N_T \times 1}$ and $c_R = (c_{R,1}, \ldots, c_{R,N_R})^T \in \mathbb{C}^{N_R \times 1}$ are given by $c_T = (N_T)^{-1/2} F_{N_T}^H \sigma_T$ and $c_R = (N_R)^{-1/2} F_{N_R}^H \sigma_R$, respectively. Because $\sigma_T$ and $\sigma_R$ are spreading sequences, after the IDFT, the randomness will remain in $c_T$ and $c_R$, respectively. In addition, $C_T$ and $C_R$ are both unitary matrices because $\sigma_T$ and $\sigma_R$ are unimodular sequences, i.e., $|\sigma_T,\ell| = 1$ and $|\sigma_R,\ell| = 1$ for $\ell = 1, \ldots, N_T$.

Let $\Phi_U \in \mathbb{C}^{M \times N}$ denote the equivalent measurement matrix for (22), where $N = N_T N_R$ since $G_T = N_T$ and $G_R = N_R$. Then, $\Phi_U$ can be written as

$$\Phi_U = R_{\Omega_G} \left( (C_T^T \otimes C_R^H) \right) = R_{\Omega_G} \left( U_G \right).$$ \quad (23)

where $U_G \in \mathbb{C}^{N \times N}$ is a transform matrix with the unitary and block-circulant properties, as shown in the following lemma.

Lemma 1 ($U_G$ is a unitary block-circulant matrix): Let $C_T = \text{Circ}(c_T)$ and $C_R = \text{Circ}(c_R)$. Suppose that $C_T$ and $C_R$ are both unitary matrices; then $U_G = C_T^T \otimes C_R^H$ is a unitary block-circulant matrix, which can be expressed as

$$U_G = \begin{bmatrix}
C_{G,1} & C_{G,N_T} & \cdots & C_{G,2} \\
C_{G,2} & C_{G,1} & \cdots & C_{G,3} \\
\vdots & \vdots & \ddots & \vdots \\
C_{G,N_T} & C_{G,N_T-1} & \cdots & C_{G,1}
\end{bmatrix},$$ \quad (24)

where $C_{G,\ell} = c_{T,\ell} c_{R,\ell}$ for $\ell = 1, \ldots, N_T$. The proof can be found in Appendix A.

According to (23), we can interpret $\Phi_U$ as a random convolutional measurement process comprised of random convolution followed by random subsampling:

- Random convolution: This step corresponds to multiply the VAD channel vector $\tilde{h}_\omega$ with $U_G$, and we have

$$\tilde{h}_\omega = U_G \tilde{h}_\omega,$$ \quad (25)

where $\tilde{h}_\omega \in \mathbb{C}^{N \times 1}$ denotes the transformed VAD channel vector. To explain intuitively the action of $U_G$, we rewrite (25) as

$$\tilde{h}_\omega = \text{vec} (C_T^H \tilde{H}_\omega C_R).$$ \quad (26)

We can see that each row vector in $\tilde{H}_\omega$ is first convolved with the sequence $c_T$ via $C_T$. Due to the randomness in $c_T$, the highly localized information of propagation paths in the limited scattering channel is spread over the entire VAD spectrum along the transmit virtual angle grid after the convolution, as illustrated in Fig. 4. Each column vector in the convolved VAD channel, i.e., $\tilde{H}_\omega C_T$, is then convolved with the sequence $c_R$ via $C_R$. In a similar fashion, the information of the propagation paths will be spread over the entire VAD spectrum along the receive virtual angle grid. As a result, by applying the circular convolutions via $C_T$ and $C_R$, no matter where AoDs and AoAs corresponding to the propagation paths in the physical channel are, the information is uniformly spread out; in other words, we can ensure that each measurement on the transformed VAD channel vector $\tilde{H}_\omega$ must contain an approximately equal amount of information from each propagation path in the channel.

Fig. 4. Illustration of applying the random convolution $C_T$ to a VAD channel with a limited scattering effect.
**Random subsampling:** This step corresponds to multiplying the transformed VAD channel vector $\mathbf{h}_c$ with $\mathbf{R}_U \mathbf{U}_G$, which denotes a subsampling at locations indexed by $\mathbf{U}_G$. Thus, the action of $\mathbf{R}_U \mathbf{U}_G$ picks up $M$ measurements out of $N$ entries of $\mathbf{h}_c$, uniformly at random. Since the information is uniformly preserved in each entry of $\mathbf{h}_c$, we can obtain a stable recovery of $\mathbf{h}_c$ from those subsampled measurements on $\tilde{\mathbf{h}}_c$.

The random convolutional measurement process described above is one class of compressed sensing tools [36]–[38], and it has been proved that this process can achieve comparable recovery performance to that of the full random measurement process. In our design, this process is further extended to a Kronecker system.

### D. Theoretical Bound for Stable Channel Recovery

To derive the theoretical bound for stable recovery from the random convolutional measurement process of $\Phi_U = \mathbf{R}_U \mathbf{U}_G$, we adopt the RIP bound of the subsampled unitary matrix.

**Theorem 1 (RIP of the subsampled unitary matrix [46]):** Suppose that an $M \times N$ measurement matrix $\Phi$ is a subsampled unitary matrix, i.e., $\Phi = \mathbf{R}_U \mathbf{U}$, where $\mathbf{U}$ is an $N \times N$ unitary matrix, and $\mathbf{R}_U$ is a subsampling operator for downsampling $M$ samples out of $N$ ones uniformly at random. Then, $\Phi$ satisfies the $(K, \delta)$--RIP for $\delta \in (1, 0)$ in a high probability if

$$M \geq \Omega(\delta^{-2} \mu(U) N K \log^2 K \log N).$$

(27)

The parameter $\mu(U)$ is defined as follows.

**Definition 2 (coherence of the unitary matrix [47]):** The coherence of an $N$-dimensional unitary matrix $\mathbf{U}$ describes the maximum magnitude of the entry in $\mathbf{U}$, i.e.,

$$\mu(U) = \max_{1 \leq p,q \leq N} |U^{p,q}|.$$

(28)

The coherence has values in the range $N^{-1/2} \leq \mu(U) \leq 1$.

Using Theorem 1, we can obtain the RIP bound of $\Phi_U$

$$M \geq \Omega(\delta^{-2} \mu(U) N K \log^2 K \log N).$$

(29)

In (29), we can see that the required number of measurements for stable channel recovery is dictated by $\mu(U_G)$. According to the properties of $U_G$ stated in Lemma 1, we have

$$\mu(U_G) = \max_{1 \leq k \leq N_T} \mu(C_G, k)$$

$$= \max_{1 \leq k \leq N_T} \mu(C_T, k)$$

$$= \max_{1 \leq k \leq N_T} \left( \left| c_{T, k} \cdot c_{R, k}^* \right| \right)^{1/2}$$

$$= \max_{1 \leq k \leq N_T} \left( \left| c_{T, k} \right| \left| c_{R, k} \right| \right)^{1/2}$$

$$= \mu(C_T) \mu(C_R).$$

(30)

From (30), the coherence of $U_G$ can be decomposed by the product of the coherences of $C_T$ and $C_R$. The coherences of the circulant matrices constructed from different unimodular spreading sequences have been investigated in [46]. Table II lists the three widely used spreading sequences, along with the corresponding $N$ and $\mu(C)$, where $C = F_N^H \text{diag}(\sigma) F_N$.

Table II

<table>
<thead>
<tr>
<th>$\sigma$</th>
<th>$N$</th>
<th>$\mu(C)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ZC sequence</td>
<td>Arbitrary</td>
<td>$(1/\sqrt{N})^2$</td>
</tr>
<tr>
<td>ML sequence</td>
<td>$2^n - 1$, $n \in \mathbb{N}$</td>
<td>$(1/\sqrt{N + 1/\sqrt{N}})^2$</td>
</tr>
<tr>
<td>Golay sequence</td>
<td>$2^n \cdot 10^m \cdot 26^n$, $n_1, n_2, n_3 \in \mathbb{N}$</td>
<td>$(2/\sqrt{N})^2$</td>
</tr>
</tbody>
</table>

Compared to the optimal bound offered by the full random design, there is an extra $\log^2 K$ factor in the structured random design. As we know, owing to the limited scattering mmWave channels, the channel sparsity level is far smaller than the channel dimension, such that $K \ll N$. Thus, the extra factor $\log^2 K$ is almost negligible. Note that although the RIP bound (31) is only derived for $G_T = N_T$ and $G_R = N_R$, it will be shown in the simulation that the structured random sensing codebook design has comparable performance to that of the full random design for arbitrary resolutions. Moreover, when the virtual angle grids with high resolutions are adopted, i.e., $G_T > N_T$ and $G_R > N_R$, the quantization error caused by the discrete channel approximation can be effectively reduced [45].

### IV. STRUCTURED RANDOM SENSING CODEBOOK DESIGN WITH POWER CONCENTRATION CAPABILITY

In the sectorized cell [29]–[31], a LSAS should only measure and estimate the channel within a local transmit angle coverage. In this scenario, if the CCS adopts the sensing beams with full angle coverages, most of the power will be wasted in the directions out of the intended angle coverage. To address this issue, based on the previously proposed dual-stage codebook structure, we propose a transmit sensing codebook with the capability of power concentration. Note that if the receive sensing codebook also needs to concentrate its power in a local angle coverage, it can be designed in a similar fashion.

#### A. Structured Random Sensing Codebook Design for Local Angle Coverage

To concentrate the power in a local angle coverage, both the sampling beamformer and the beam spreader of the transmit sensing beam should be specifically designed. As shown in Fig. 5, the $m$-th transmit sensing beam $p_m = \Delta_T w_{T,m}$
in the structured random sensing codebook is constructed as follows:

- **Sampling beamformer**: In this stage, the sampling beam $w_{T,m}$ should be selected from a candidate DFT codebook rather than a full DFT codebook. Consider that the candidate DFT codebook consisting of $N_{DFT}^T$ DFT beams that cover the local angle coverage, the indices of these DFT beams is defined in a subsequence $\Gamma_T \subset \{1, \ldots, N_T\}$, $|\Gamma_T| = N_{DFT}^T < N_T$, which is also regarded as the intended sampling region. Then, $w_{T,m}$ can be selected from $F_{N_T}^T$ through the sampling index $\Omega_{T,m}$, as illustrated by the corresponding beam pattern (blue) in Fig. 5. Thus, the sampling beam can be expressed as

$$w_{T,m} = F_{N_T}^T r_{T,m},$$  

(32)

where $r_{T,m} \in \mathbb{R}^{1 \times N_T}$ is the sampling vector for selecting the columns of $F_{N_T}^T$ indexed by $\Omega_{T,m}$.

- **Beam spreader**: In this stage, differing from the design in Section III-A, the beam spreader $\Delta_T$ should spread the sampling beam $w_{T,m}$ over a limited angle coverage instead of a full angle coverage. Consider a unimodular sequence with good autocorrelation property $\alpha = [\alpha_1, \ldots, \alpha_{N_C}]^T \in \mathbb{C}^{N_C \times 1}$. The length of $\alpha$ should be a factor of $N_T$, i.e., $N_T = D_T N_C$, where $D_T$ is a positive integer. Then, the spreading sequence $\sigma_T$ in the beam spreader $\Delta_T$ is constructed by

$$\sigma_{T,\ell} = \alpha_{\ell \cdot (\ell - 1) / D_T + 1},$$  

(33)

for $\ell = 1, \ldots, N_T$. Throughout this paper, we define the spreading factor $S_T \in \mathbb{Z}$ of $\sigma_T$ as

$$S_T = \frac{N_C}{N_T}.$$  

(34)

Similar to the direct-sequence spread spectrum (DSSS), where the expansion of signal bandwidth after the spreader is varied with different spreading factors, the expansion of beamwidth after the beam spreader $\Delta_T$ can be configured by adjusting $S_T$. When $S_T = 1$ (i.e., $N_C = N_T$), the resultant sensing beam exhibits a full angle coverage, as illustrated in Fig. 6(a), which has a relatively low directivity gain under a fixed transmit power. In contrast, when $S_T < 1$ (i.e., $N_C < N_T$), the beamwidth of the resultant sensing beam is narrower as $S_T$ decreases, and its power is concentrated in the direction of corresponding sampling beam to gather a larger directivity gain, as illustrated in Fig. 6(b) and Fig. 6(c). However, when $S_T$ is too small, the beam spreader has limited expansion of beamwidth, the resultant sensing beam cannot achieve an effective measurement for stable recovery. Therefore, in Section IV-C, we will discuss the design of $S_T$ for a local angle coverage.

To generate a transmit structured random sensing codebook consisting of $M$ sensing beams for a local angle coverage, these sensing beams are constructed by $M$ sampling beams selected from the candidate DFT codebook $F_{N_T}^T$ and a same beam spreader adopting the spreading sequence with $S_T < 1$. Thus, the transmit sensing codebook can be expressed as

$$P = \Delta_T F_{N_T}^T R_{\Omega_T}^T,$$  

(35)

where the sampling index set $\Omega_T$ for $R_{\Omega_T}$ is also derived from (17). However, since the size of the transmit DFT codebook is reduced, the subsampling index set $\Omega_G$ for $R_{\Omega_G}$ in (17) should be downsampled from $\{1, \ldots, N\}$, where $N = N_{DFT}^R N_R < N$, uniformly at random instead of $\{1, \ldots, N\}$.

**B. Equivalent Measurement Process of the Structured Random Compressed Channel Sensing for Local Angle Coverage**

By adopting the transmit sensing codebook (35) at the BS, we can rewrite (16) as

$$y = \sqrt{\rho} R_{\Omega_G} \left( (\Delta_T F_{N_T}^T)^T \otimes (\Delta_R F_{N_R})^H \right) \text{vec}(H) + e.$$  

(36)
Since the AoDs must locate in an intended angle coverage, we can substitute the VAD representation (4) into (36). Thus,

$$y \approx \sqrt{\rho} R_G \left( \left( \Delta_T F^T_{N_Y} \right)^T \otimes \left( \Delta_R F^H_{N_R} \right)^H \right)$$

$$\times \text{vec} \left( A_R \hat{H}_{Q_T} \left( A^H_{Q_T} \right)^H \right) + e$$

$$\approx \sqrt{\rho} R_G \left( \left( \left( A^H_{Q_T} \right)^H \Delta_T F^T_{N_Y} \right)^T \otimes \left( A^H_R \Delta_R F^H_{N_R} \right)^H \right)$$

$$\times \text{vec} \left( \hat{H}_{Q_T} \right) + e$$

$$\approx \sqrt{\rho} R_G \left( \left( \left( A^H_{Q_T} \right)^H \Delta_T F^T_{N_Y} \right)^T \otimes \left( A^H_R \Delta_R F^H_{N_R} \right)^H \right)$$

$$\times \hat{H}_{Q_T} + e,$$

(37)

where $\hat{H}_{Q_T} = \text{vec} \left( \hat{H}_{Q_T} \right) \in \mathbb{C}^{|Q_T| G_M \times 1}$ is the VAD channel vector with the constraint on the AoD. For simplicity, we consider that $G_T = N_Y$ and $G_R = N_R$. In this case, we have $Q_T = \Gamma_T$, and (37) can be rewritten as

$$y \approx \sqrt{\rho} R_G \left( \left( F^T_{N_Y} \right)^T \otimes \left( F^H_{N_R} \Delta_R F^H_{N_R} \right)^H \right) \hat{h}_{g_T} + e$$

$$\approx \sqrt{\rho} R_G \left( \left( \left( C^T_{Q_T} \Gamma_T \right)^T \otimes C^H_R \right)^H \right) \hat{h}_{g_T} + e$$

$$\approx \sqrt{\rho} R_G \left( T_G \hat{h}_{g_T} \right) + e,$$

(38)

where $R_G \in \mathbb{R}^{M \times N}$ denotes the subsampling operator for downselecting $M$ samples out of $\hat{N}$ according to $Q_T$, and $T_G \in \mathbb{C}^{N \times N}$ denotes the transform matrix.

As mentioned previously, before the subsampling by $R_G$, the transform by $T_G$ on the VAD channel is the key action to achieve stable channel recovery from (38). Let $\hat{h}_{g_T} \in \mathbb{C}^{N \times 1}$ denote the transformed $\hat{h}_{g_T}$; then it can be expressed as

$$\hat{h}_{g_T} = T_G \hat{h}_{g_T}$$

$$= \text{vec} \left( C^H_R \hat{H}_{Q_T} \left( C^H_R \right)^T \right) \hat{h}_{g_T}$$

$$= \text{vec} \left( C^H_R \left( \hat{H}_{Q_T} C_{Q_T} \right)^T \right),$$

(39)

After the transform by $T_G$, the information of each propagation path in $\hat{h}_{g_T}$ is spread over the VAD spectrum along the transmit and receive virtual angle grids as broadly as possible. We can see that the difference between the transforms of (39) and (26) is that the convolved VAD channel $\hat{H}_{Q_T} C_{Q_T}$ is restricted within a limited sampling region $\Gamma_T$ on the transmit virtual angle grid rather than the entire VAD spectrum. This is because we have prior knowledge that the information must locate in the intended sampling region $\Gamma_T$, as illustrated in Fig. 7(a), and the search space on the VAD spectrum can be reduced. When $C_{Q_T}$ is constructed by $S_{Q_T}$ with $S_{Q_T} = 1$, because the information is spread over the entire VAD spectrum by $C_{Q_T}$, stable recovery from the subsampled $\hat{h}_{g_T}$ can be reasonably achieved. On the other hand, when $C_{Q_T}$ is constructed by $S_{Q_T}$ with $S_{Q_T} < 1$, even the effect of information expansion by $C_{Q_T}$ is limited, stable recovery still can be guaranteed for the CCS in the local angle coverage. This is because, in (39), we only have to ensure that each measurement on the convolved VAD channel within the intended sampling region, i.e., $\hat{h}_{g_T}$, should preserve an approximately equal amount of the information. As illustrated in Fig. 7(b), when the information is only spread within the intended sampling region $\Gamma_T$ rather than the entire VAD spectrum, which can be achieved by choosing an appropriately small $S_{Q_T}$, channel recovery from the subsampled $\hat{h}_{g_T}$ still can perform stable embedding. On the other hand, although choosing a small $S_{Q_T}$ for a sensing codebook can more concentrate the power to gather larger directivity gains, it should be sufficiently large to ensure the effect of information expansion in the intended sampling region.

C. Theoretical Bound for Stable Channel Recovery

According to the definition of $T_G = \left( C^T_{Q_T} \Gamma_T \right)^T \otimes C^H_R$, it can be rewritten as a square submatrix of $U_G$, such that

$$T_G = \left( C^T_{Q_T} \otimes C^H_R \right)^{A_T}$$

$$= U_G^{A_T},$$

(40)

where $A_T$ is a subsequence of $\{1, \ldots, \hat{N}\}$ with the length of $N_{DFT} N_R$ for collecting the block of the entries in $U_G$ whose row and column are indexed by

$$\Lambda_T = \left[ (\Gamma_{T,1} - 1) N_R + 1, (\Gamma_{T,1} - 1) N_R + 2, \ldots, \Gamma_{T,1} N_R, \right.$$

$$\ldots, (\Gamma_{T,N_{DFT}} - 1) N_R + 1, (\Gamma_{T,N_{DFT}} - 1) N_R + 2, \ldots, \Gamma_{T,N_{DFT}} N_R \left. \right] .$$

(41)
Let  $\Phi_T \in \mathbb{C}^{M \times N}$ denote the equivalent measurement matrix for (38). It can be expressed as

$$
\Phi_T = R_{\Omega_G} T_G
= R_{\Omega_G} U_{G^{T,\Lambda_T}}^T.
$$

(42)

Since $T_G$ is embedded into $U_G$, we can derive an analogous bound of $\Phi_T$ from the RIP bound of $\Phi_U$ such that

$$
M \geq O(\delta^{-2} \mu^2(T_G) N K \log^2 K \log N),
$$

(43)

where

$$
\mu(T_G) = \begin{cases}
\mu(U_G), & \mu(U_G) > N^{-1/2}; \\
(N)^{-1/2}, & \mu(U_G) \leq N^{-1/2}.
\end{cases}
$$

(44)

In (30), we know that $\mu(U_G) = \mu(C_T)\mu(C_R)$. When $C_T$ is constructed by $\sigma_T$ with $S_T = 1$, since $\mu(U_G) = N^{-1/2}$ is always smaller than $N^{-1/2}$, we have the RIP bound of $\Phi_T$

$$
M \geq O(\delta^{-2} K \log^2 K \log N).
$$

(45)

On the other hand, when $C_T$ is constructed by $\sigma_T$ with $S_T < 1$, the ideal coherence of $C_T$ can be given in the following lemma.

**Lemma 2** (ideal coherence of $C_T$ when $S_T < 1$): Let $C_T = \text{Circ}(c_T)$, where $c_T = (N_T)^{-1/2} F_N^T \sigma_T$, and $\sigma_T$ is constructed by a sequence $\alpha$ with the length of $N_C < N_T$ using (33). If $\alpha$ is a unimodular sequence with the zero autocorrelation property, then $C_T$ has the ideal coherence $\mu(C_T) = (N_C)^{-1/2}$. The proof can be found in Appendix B.

From Table II and Lemma 2, it can be seen that when both $\alpha$ (for constructing $\sigma_T$) and $\sigma_R$ are the ZC sequences, we have $\mu(U_G) = \mu(C_T)\mu(U_R) = (N_C N_R)^{-1/2}$. Thus, the RIP bound of $\Phi_T$ can be derived

$$
M \geq \begin{cases}
O(\delta^{-2}(N_{DFT}^L/N_C) K \log^2 K \log N), N_{DFT}^L > N_C; \\
O(\delta^{-2} K \log^2 K \log N), N_{DFT}^L \leq N_C < N_T.
\end{cases}
$$

(46)

We can see that when $N_{DFT}^L \leq N_C < N_T$, i.e., $1 > S_T \geq N_{DFT}^L / N_T$, $\Phi_T$ provides a bound the same with (45). However, when $N_{DFT}^L > N_C$, i.e., $N_{DFT}^L / N_T > S_T$, more measurements are required to guarantee stable channel recovery. This theoretical result agrees with our observation in Section IV-B.

According to (46), the design of spreading factor for a local angle coverage can be derived. By choosing an appropriately small spreading factor satisfying $S_T \geq N_{DFT}^L / N_T$ for the transmit sensing codebook (35), it can achieve the same bound with that of the fully spread sensing codebook (i.e., $S_T = 1$) while concentrating the power to gather larger directivity gains. On the other hand, the design of spreading factor is also dictated by the SNR. As we know, the RIP bound (46) only holds when the minimum requirement on the ratio of signal power after measurement to noise power is satisfied [35]. Therefore, when the SNR is not sufficiently high, to improve the robustness of channel recovery, we should choose a spreading factor smaller than $N_{DFT}^L / N_T$ to more concentrate the power of the sensing codebook.

V. Simulation Results

We evaluate the performance of the proposed sensing codebook design for the CCS through several simulations. In our simulations, the system and channel models described in Section II are set as follows:

- The BS and UE are equipped with $N_T = 128$ and $N_R = 32$ antennas, respectively.
- The number of clusters in the physical channel is set $K = 6$.
- The physical AoDs are uniformly distributed in a local AoD coverage, such that $\{\cos(\theta_k^{4\alpha})\}_{k=1,...,K} \in [-1/3, 1/3]$.
- The physical AoAs are uniformly distributed in a full AoA coverage, such that $\{\cos(\theta_k^{4\alpha})\} = 1,...,K \in [-1, 1]$.
- Channel recovery is performed using the orthogonal matching pursuit (OMP) algorithm adopted in [45].

The normalized mean square error (NMSE) is adopted as the recovery performance metric, which is defined as

$$
\text{NMSE} = 10 \log \left( \mathbb{E} \left[ \| \tilde{H} - H \|_F^2 / \| H \|_F^2 \right] \right).
$$

(47)

We compare the recovery performance of channel recovery from CCS that adopts the following different codebooks:

1) **Exhaustive DFT codebook (EXH-DFT):** Both BS and UE adopt the DFT codebooks to exhaustively measure the channel. At the BS, $F_{N_T}^T$ with $N_{DFT}^T = 44$ is adopted to cover the local AoD coverage. At the UE, $F_{N_R}^T$ is adopted to cover the full AoA coverage.

2) **Full random Sensing codebook (FR-CS):** At the BS and the UE, full random sensing codebooks are adopted, where the transmit and receive sensing beams are drawn from the i.i.d. Bernoulli bipolar random variables, respectively.

3) **Structured random sensing codebook (SR-CS) with $S_T = 1$:** At the BS and the UE, the spreading codebooks (15) are adopted, where the spreading sequences are ZC sequences with the lengths of 128 and 32, respectively.

4) **SR-CS with $S_T = 1/4$:** At the BS, the transmit sensing codebook with power concentration (35) is adopted, where the spreading sequence is constructed by the ZC sequence with the length of 32 using (33), and the sampling beams are selected from the DFT codebook $F_{N_T}^T$ with $N_{DFT}^T = 44$. At the UE, the receive sensing codebook is the same as 3).

5) **SR-CS with $S_T = 1/8$:** At the BS, the transmit sensing codebook with power concentration (35) is adopted, where the spreading sequence is constructed by the ZC sequence with the length of 16 using (33), and the sampling beams are selected from the DFT codebook $F_{N_T}^T$ with $N_{DFT}^T = 44$. At the UE, the receive sensing codebook is the same as 3).

Each simulation result is obtained from Monte Carlo trials over 5,000 independent channel realizations.

A. NMSE Under Various SNRs

Fig. 8 depicts the recovery performance under various SNRs with $M = 320$, which is approximately 23% of that in the
EXH-DFT. We also consider two different resolution settings, \( \{G_T, G_R\} = \{128, 32\} \) and \( \{G_T, G_R\} = \{512, 128\} \).

We can first observe that all the codebooks exhibit significant NMSE improvements when the virtual angle grids with higher resolutions are adopted. This is because when the impact of quantization error is reduced, the recovery performance of the CCS can be effectively enhanced. Next, we can observe that the SR-CS with \( S_T = 1 \) has comparable recovery performance to that of the FR-CS. However, both of them exhibit huge performance gaps compared to the EXH-DFT, especially in low SNR. This is because the power of the fully spread sensing beams are not sufficient for robust channel recovery in low-SNR regimes. Fortunately, we can adopt the SR-CS with power concentration to enhance the robustness. As we can see, when \( S_T = 0 \), the SR-CS with \( S_T = 1/8 \) exhibits approximately 2 dB and 4 dB performance gains over the FR-CS and SR-CS with \( S_T = 1 \) in the two different resolution settings, respectively. The performance gap between the SR-CS with \( S_T = 1/8 \) and the EXH-DFT is less than 2 dB in any SNR.

**B. NMSE Under Various Numbers of Measurements**

Fig. 9 illustrates the recovery performance under various numbers of measurements when \( SNR \in \{0 \text{dB}, 10 \text{dB}\} \). In this simulation, we only consider the transmit and receive virtual angle grids with \( \{G_T, G_R\} = \{512, 128\} \) to well approximate the physical channel. As the EXH-DFT has full measurements from the transmit and receive DFT codebooks, its recovery performance remains constant for different values of \( M \).

We can observe that the SR-CS with \( S_T = 1 \) has comparable recovery performance to that of the FR-CS with any number of measurements. However, both of them are far worse than the EXH-DFT, especially in low SNR. To reduce the performance gaps, we can adopt the SR-CS with power concentration. As we can see, when \( M \geq 200 \), the SR-CS with \( S_T = 1/8 \) exhibits more than 4 dB and 2 dB NMSE improvements over the SR-CS with \( S_T = 1 \) for \( SNR = 0 \) dB and 10 dB, respectively. It is worth noting that the improvement from the power concentration is more significant when the SNR is lower.

**C. Achievable Rate Under Various Numbers of Measurements**

Fig. 10 depicts the achievable rates given by the estimated channels, which is the final aim of channel estimation, under various numbers of measurements when \( SNR = 0 \) dB and \( \{G_T, G_R\} = \{512, 128\} \). In this simulation, the achievable rate is derived from the optimal unconstrained precoder [40] for the hybrid AD LSAS with a limited number of RF chains

\[
R = \log_2 \det \left( I_{N_S} + \frac{\rho}{N_{RF}} R^{-1} W_{opt} H F_{opt} H^H W_{opt} \right),
\]

where the unconstrained precoder \( F_{opt} \in \mathbb{C}^{N_T \times N_{RF}} \) and combiner \( W_{opt} \in \mathbb{C}^{N_R \times N_{RF}} \) are designed by selecting the \( N_{RF} \)-dominant singular vectors of the estimated channel, \( R = \sigma^2_n W_{opt} H W_{opt} \) denotes the noise covariance matrix after combining, and \( N_{RF} \) denotes the number of RF chains.

We can observe that when \( N_{RF} = 4 \), the SR-CS with \( S_T = 1/8 \) achieves 95% optimal rate by employing only 140 measurements, which saves approximate 90% sweeping overhead from the EXH-DFT with negligible rate losses. In contrast, the FR-CS and the SR-CS with \( S_T = 1 \) exhibit significant rate losses from the EXH-DFT. Even more measurements are employed for them, their performance saturates without any improvement. This is because channel recovery has the minimum requirement on the ratio of signal power after measurement to noise power, which is scaled as \( N \) regardless of \( M \) [35]. When both SNR and
directivity gains are not sufficiently high, robust channel recovery cannot be achieved, and more measurements cannot reduce the performance gap.

D. NMSE Under Various Spreading Factors

Fig. 11 depicts the recovery performance of the SR-CS with various $S_T$ when $M = 140$, \( \{G_T, G_R\} = \{512, 128\} \), and SNR $\in \{-5 \text{ dB}, 0 \text{ dB}, 10 \text{ dB}, 20 \text{ dB}, 30 \text{ dB}\}$.

We can observe that when SNR $= 30$ dB, the SR-CS with $S_T = 1/2$ has the same performance with that of the SR-CS with $S_T = 1$. This is because the $S_T = 1/2$ satisfies $S_T \geq N_{DFT}^2/N_T = 44/128$, which is sufficiently large to ensure the effect of information expansion in the intended sampling region. However, the SR-CS with spreading factor smaller than $N_{DFT}^2/N_T = 44/128$ (i.e., $S_T = 1/4, 1/8, 1/16, 1/32$) suffers performance degradation since the measurements from the structured random CCS becomes ineffective for channel recovery. When SNR decreases, the performance of SR-CS with $S_T = 1$ becomes worse than that of the SR-CS with $S_T < 1$. Meanwhile, the spreading factor of the SR-CS achieving the lowest NMSE becomes smaller as the SNR decreases, even if it is smaller than $N_{DFT}^2/N_T = 44/128$. This because when the SNR is not sufficiently high, the larger directivity gains of the sensing codebook can increase the ratio of signal power after measurement to noise power to improve the performance. Thus, in the low-SNR regimes, we need the SR-CS with spreading factor smaller than $N_{DFT}^2/N_T = 44/128$ to more concentrate the power of each sensing beam.

E. NMSE Under Various Resolutions

Fig. 12 shows the recovery performance under various resolutions of the virtual angle grids when $M = 320$ and SNR $\in \{0 \text{ dB}, 10 \text{ dB}\}$. In this simulation, we set the resolutions to be scaled with the factor $r$, such that $G_T = rN_T$ and $G_R = rN_R$.

We can observe that the SR-CS with $S_T = 1$ has comparable recovery performance to that of the FR-CS when $G_T > N_T$ and $G_R > N_R$ even when we only offer the RIP bound for $G_T = N_T$ and $G_R = N_R$. Furthermore, as we expect, when the resolutions increase with $r$, the recovery performance of each codebook is effectively enhanced. It comes from the fact that the error caused by quantizing the physical AoDs/AoAs in fixed virtual angles is reduced by increasing the resolutions of virtual angle grids. On the other hand, with higher resolutions, we can see that the SR-CSs with power concentration exhibit a larger performance gain over the sensing codebooks with full angle coverages.

VI. CONCLUSION

In this paper, a new class of CCS for the mmWave LSAS was presented. First, we proposed a structured random sensing codebook design. Owing to its structured nature, instead of signaling or storing the explicit sensing codebook, the configuration of the proposed sensing codebook can be implicitly represented to reduce the signaling or storage overhead. For the CCS adopting the proposed sensing codebook design, we offered an RIP bound derived from the equivalent measurement process, and showed that the proposed structured random design compares favorably to the full random design. Second, based on the proposed dual-stage codebook structure, we further developed a sensing codebook with the capability of power concentration. For CCS in a local angle coverage, we concentrated the sensing beams in a local angle coverage, we concentrated the sensing beams in the local angle coverage to increase directivity gains. Thus, the stringent requirement on the SNR for robust channel recovery could be alleviated. Simulation results showed that the proposed sensing codebook design has comparable performance to that of the full random design. In low-SNR regimes, the performance was substantially enhanced by the proposed sensing codebook with power concentration.

APPENDIX A

PROOF OF Lemma 1

According to the definition of $U_G$ in Lemma 1, we have

\[
U_G = C_T^T \otimes C_R^H \\
= C_T \otimes C_R^* \\
= \begin{bmatrix}
C_{T,1}^* & C_{T,N_T}^* & \cdots & C_{T,2}^* \\
C_{T,2}^* & C_{T,1}^* & \cdots & C_{T,3}^* \\
\vdots & \vdots & \ddots & \vdots \\
C_{T,N_T}^* & C_{T,N_T-1}^* & \cdots & C_{T,1}^*
\end{bmatrix}
\]
\[
C \begin{bmatrix}
C_{G,1} & C_{G,N_T} & \cdots & C_{G,2} \\
C_{G,2} & C_{G,1} & \cdots & C_{G,3} \\
\vdots & \vdots & \ddots & \vdots \\
C_{G,N_T} & C_{G,N_T-1} & \cdots & C_{G,1}
\end{bmatrix},
\]

where (a) follows from \(C_T\) and \(C_R\) are both circulant matrices, and \(C_{G,\ell} = C_{Tor}C_{R} \in \mathbb{C}^{N_R \times N_R}\) for \(\ell = 1, \ldots, N_T\). As we can see, \(U_C\) is a block-circulant matrix. On the other hand,

\[
U_C^H U_C^H = (C_T^H \otimes C_R^H)(C_T \otimes C_R)^H
= (C_T^H \otimes C_R^H)(C_T^H \otimes C_R)
= (a)(C_T^H C_T^*) \otimes (C_R^H C_R)
= (b)(I_{N_T}^*) \otimes (I_{N_R}) = I_N,
\]

where (a) follows from \((A \otimes B)(C \otimes D) = (AC) \otimes (BD)\), and (b) follows from \(C_T\) and \(C_R\) are both unitary matrices. In a similar fashion, we can also derive \(U_C^H U_C = I_N\). Thus, since \(U_C^H U_C = U_C = U_R\), \(U_C^H U_C = I_N\), \(U_C\) is a unitary matrix. Combining the results of (A1) and (A2), we have proved Lemma 1.

**APPENDIX B**

**PROOF OF LEMMA 2**

Since the \(C_T\) is a circulant matrix specified by the sequence \(c_T\), the coherence of \(C_T\) can be written as

\[
\mu(C_T) = \max_{1 \leq k \leq N_T-1} |C_T(k)|.
\]

According to the definition of \(c_T\) in Lemma 2, the \(k\)-th entry of \(c_T\) can be rewritten as

\[
c_{T,k} = \frac{1}{N_T} \sum_{\ell=1}^{N_T} \sigma_{T,\ell} e^{j \frac{2\pi}{N_T} (k-1)(\ell-1)}
= \frac{1}{N_T} \left( \sum_{m=1}^{N_C} \alpha_m e^{j \frac{2\pi}{N_C} (k-1)(m-1)} \right.
+ \sum_{m=1}^{N_C} \alpha_m e^{j \frac{2\pi}{N_C} (k-1)(m-1) + j \frac{2\pi}{N_T} k}
+ \sum_{m=1}^{N_C} \alpha_m e^{j \frac{2\pi}{N_C} (k-1)(m-1) + j \frac{2\pi}{N_T} (D_T-1)k}
\right.
= \frac{1}{N_T} \left( \sum_{m=1}^{N_C} \alpha_m e^{j \frac{2\pi}{N_C} (k-1)(m-1)} \right) \left( \sum_{n=1}^{D_T} e^{j \frac{2\pi}{D_T} (k-1)(n-1)} \right). \tag{B2}
\]

Substituting (B2) into (B1), we have

\[
\mu(C_T) = \frac{1}{N_T} \max_{1 \leq k \leq N_T} \left| \sum_{n=1}^{D_T} e^{j \frac{2\pi}{D_T} (k-1)(n-1)} \right|
\]

\[
\times \left( \sum_{m=1}^{N_C} \alpha_m e^{j \frac{2\pi}{N_C} (k-1)(m-1)} \right). \tag{B3}
\]

References


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