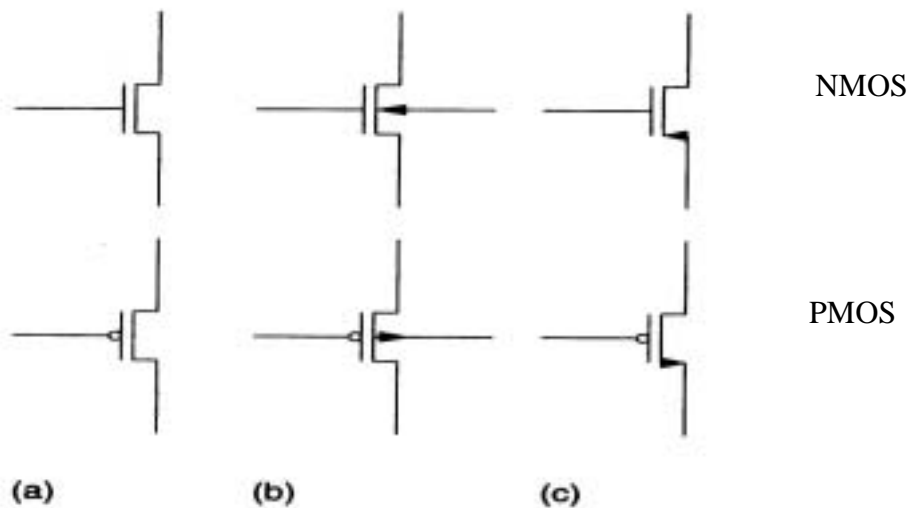


Chapter 2 MOS Transistor theory

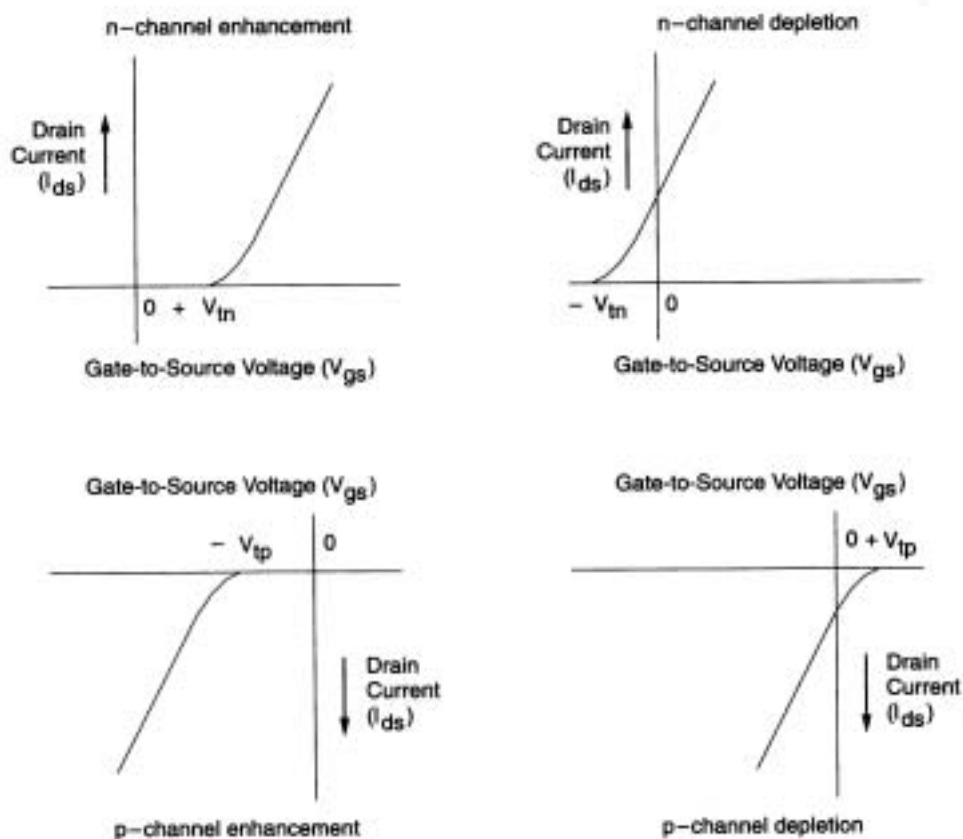
2.1 Introduction

- An MOS transistor is a majority-carrier device, in which the current in a conducting channel between the source and the drain is modulated by a voltage applied to the gate.
- Symbols



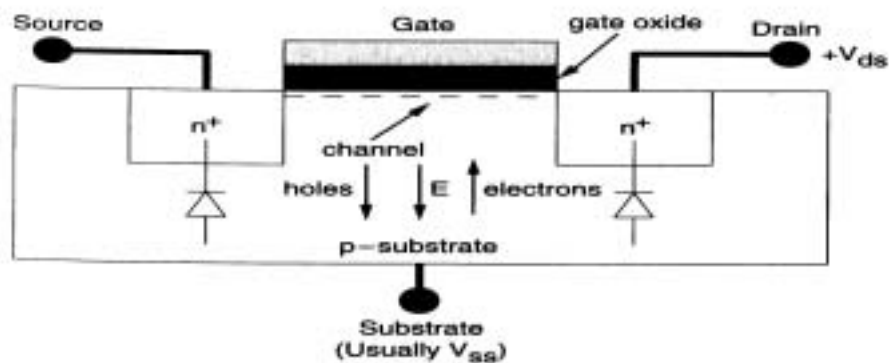
- NMOS (n-type MOS transistor)
 - (1) Majority carrier = electrons
 - (2) A positive voltage applied on the gate with respect to the substrate enhances the number of electrons in the channel and hence increases the conductivity of the channel.
 - (3) If gate voltage is less than a threshold voltage V_t , the channel is cut-off (very low current between source & drain).
- PMOS (p-type MOS transistor)
 - (1) Majority carrier = holes
 - (2) Applied voltage is negative with respect to substrate.

- Threshold voltage (V_t):
The voltage at which an MOS device begins to conduct ("turn on")
- Relationship between V_{gs} (gate-to-source voltage) and the source-to-drain current (I_{ds}), given a fixed drain-to-source voltage (V_{ds}).



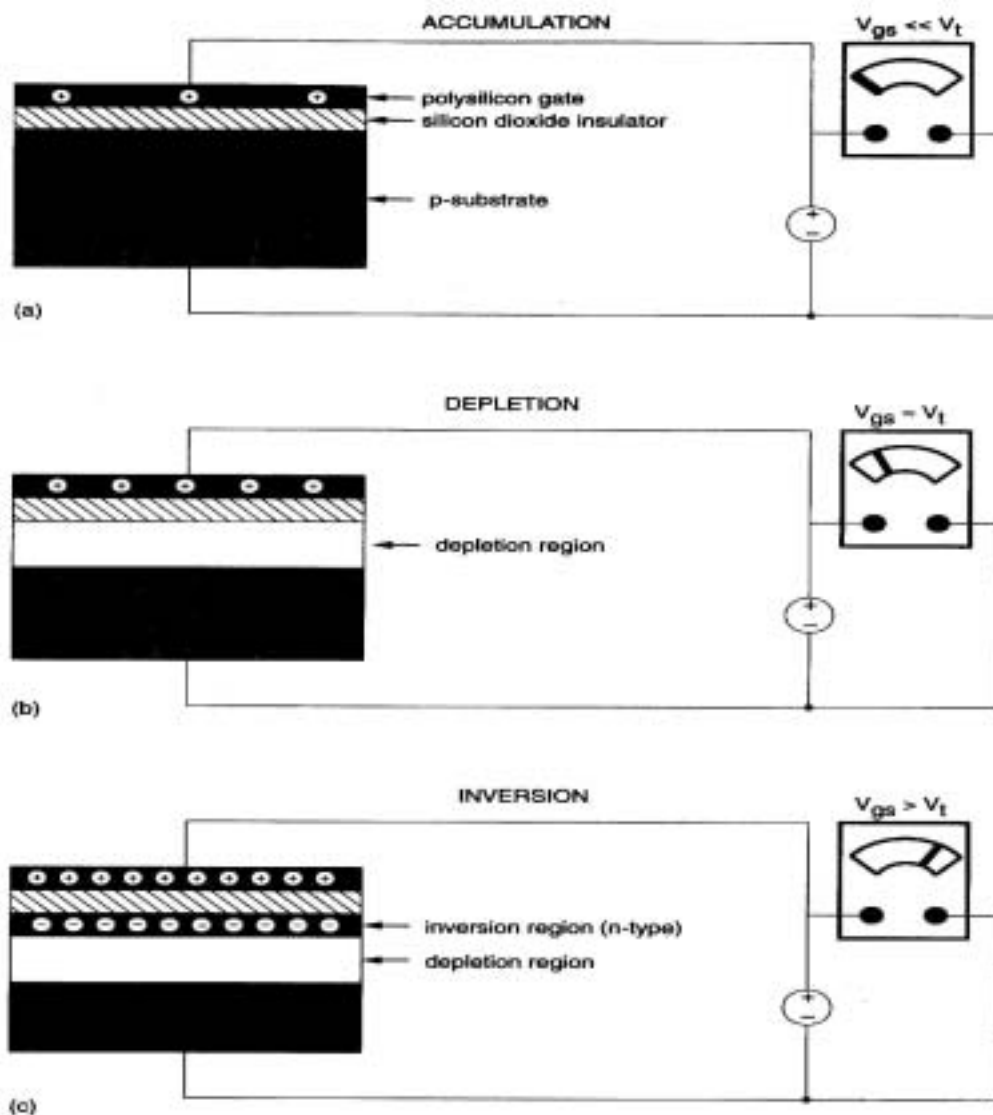
- (1) Devices that are normally cut-off with zero gate bias are classified as "enhancement-mode" devices.
- (2) Devices that conduct with zero gate bias are called "depletion-mode" devices.
- (3) Enhancement-mode devices are more popular in practical use.

2.1.1 NMOS Enhancement Transistor

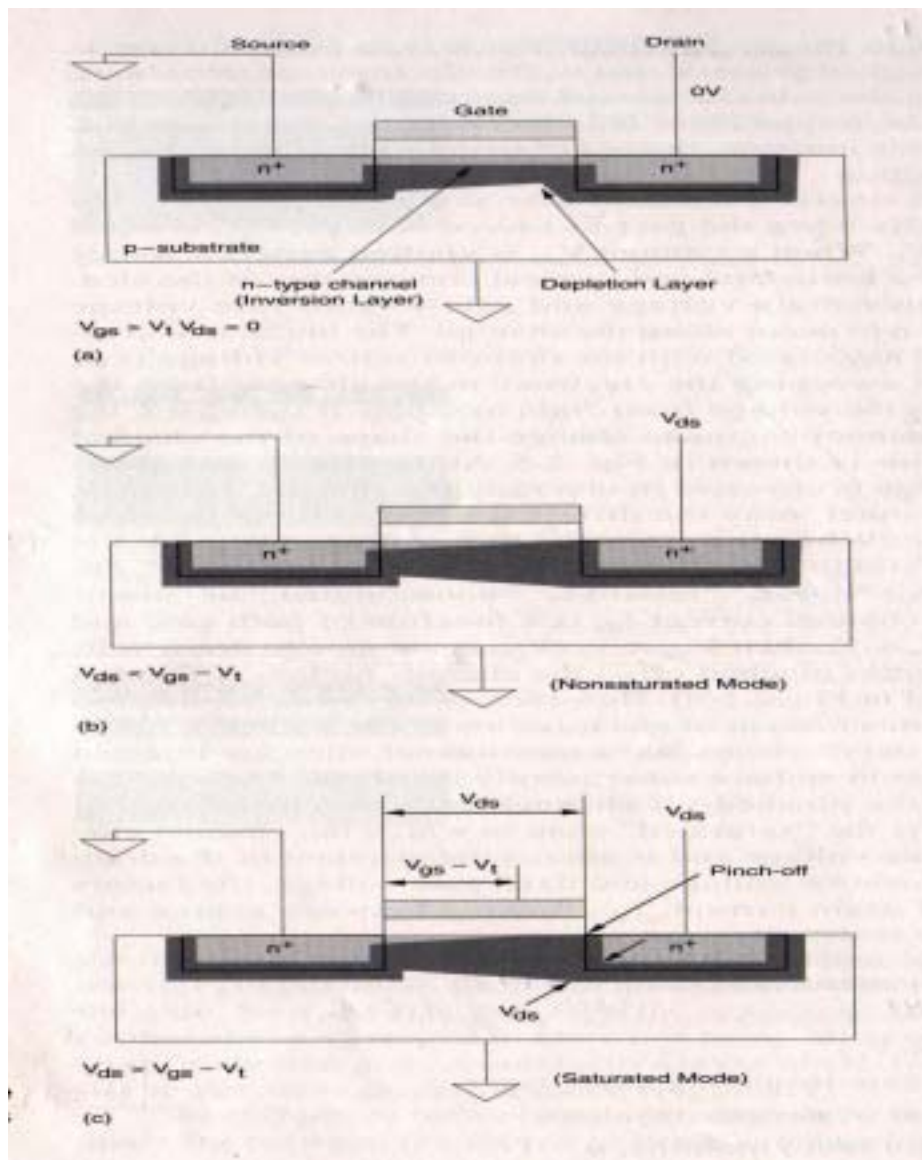


- Consist of
 - (1) Moderately doped p-type silicon substrate
 - (2) Two heavily doped n^+ regions, the source and drain, are diffused.
 - (3) Channel is covered by a thin insulating layer of silicon dioxide (SiO_2) called " Gate Oxide "
 - (4) Over the oxide is a polycrystalline silicon (polysilicon) electrode, referred to as the "Gate"
- Features
 - (1) Since the oxide layer is an insulator, the DC current from the gate to channel is essentially zero.
 - (2) No physical distinction between the drain and source regions.
 - (3) Since SiO_2 has low loss and high dielectric strength, the application of high gate fields is feasible.
- In operation
 - (1) Set $V_{ds} > 0$ in operation
 - (2) $V_{gs} = 0 \rightarrow$ no current flow between source and drain. They are insulated by two reversed-biased PN junctions (see Fig 2.3).

- (3) When $V_g > 0$, the produced E field attracts electrons toward the gate and repels holes.
- (4) If V_g is sufficiently large, the region under the gate changes from p-type to n-type (due to accumulation of attracted electrons) and provides a conducting path between source and drain. $\leftarrow \rightarrow$ The thin layer of p-type silicon is said to be "inverted".
- (5) Three modes (see Fig 2.4)
- Accumulation mode ($V_{gs} \ll V_t$)
 - Depletion mode ($V_{gs} = V_t$)
 - Inversion mode ($V_{gs} > V_t$)



- Electrically
 - (1) An MOS device can be considered as a voltage-controlled switch that conducts when $V_{gs} > V_t$ (given $V_{ds} > 0$)
 - (2) An MOS device can be considered as a voltage-controlled resistor (See Fig 2.5)
- Effective gate voltage ($V_{gs} - V_t$)
- At the source end , the full gate voltage is effective in the inverting the channel.
- At the drain end , only the difference between the gate and drain voltage is effective



- Pinch-off

- (1) $V_{ds} > V_{gs} - V_t \Rightarrow V_{gd} < V_t \Rightarrow V_d > V_g - V_t$ (V_g is not big enough)

- (2) The channel no longer reaches the drain. (Fig 2.5 c)

- (3) As electrons leave the drain depletion region and are subsequently accelerated toward the drain.

- (4) The voltage across the pinched-off region remains at $(V_{gs} - V_t)$ => "saturated" state in which the channel current is controlled by V_g , and is independent of V_d

- For fixed V_{ds} and V_g , I_{ds} is function of

- (1) Distance between drain & source

- (2) Channel width

- (3) V_t

- (4) Thickness of gate oxide

- (5) The dielectric constant of gate oxide

- (6) Carrier (hole or electron) mobility, μ .

- Conducting mode

- (1) "cut-off" region: $I_{ds} \approx 0$, $V_{gs} < V_t$

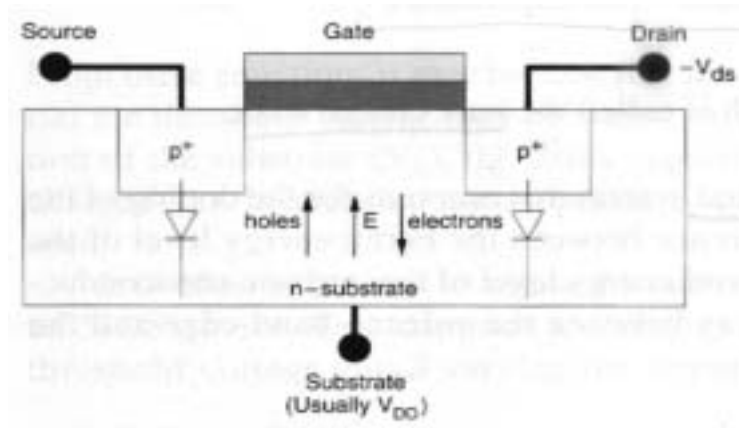
- (2) "Nonsaturated" region: weak inversion region, when I_{ds} depends on V_g & V_d

- (3) "Saturated" region: channel is strongly inverted and I_{ds} is ideally independent of V_{ds} (pinch-off region)

- (4) "Avalanche breakdown" (pinch-through): very high $V_d \Rightarrow$ gate has no control over I_{ds}

2.1.2 PMOS Enhancement Transistor

- (1) $V_g < 0$
- (2) Holes are major carrier
- (3) $V_d < 0$, which sweeps holes from the source through the channel to the drain .



2.1.3 Threshold voltage

- A function of
 - (1) Gate conductor material
 - (2) Gate insulator material
 - (3) Gate insulator thickness
 - (4) Impurity at the silicon-insulator interface
 - (5) Voltage between the source and the substrate V_{sb}
 - (6) Temperature
 - a. $-4 \text{ mV}/^\circ\text{C}$ – high substrate doping
 - b. $-2 \text{ mV}/^\circ\text{C}$ – low substrate doping

2.2 MOS equations

2.2.1 Basic DC equations

- Three MOS operating regions

(1) Cutoff or subthreshold region

$$I_{ds}=0, V_{gs} \leq V_t$$

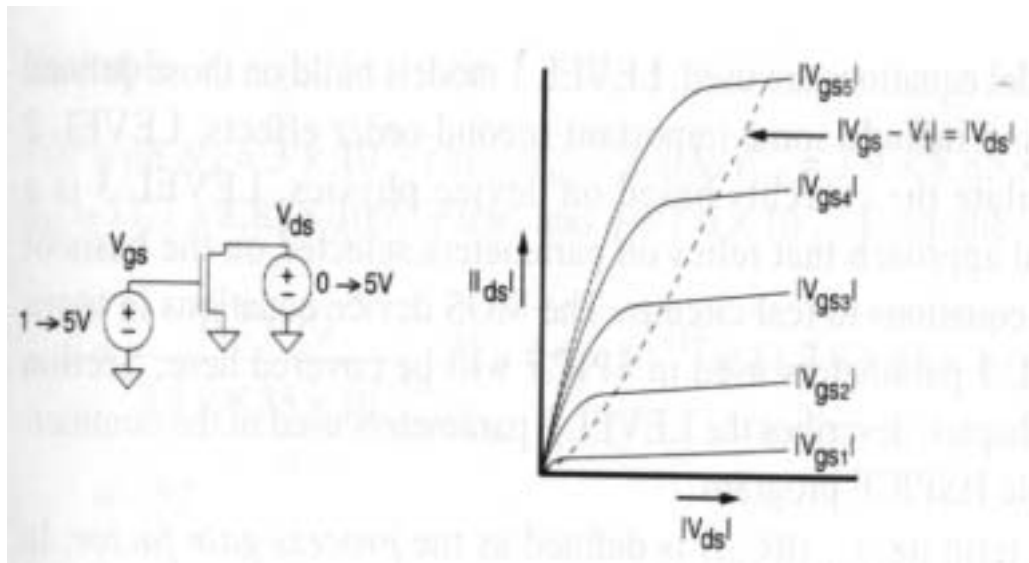
(2) Nonsaturation, linear or triode region

$$I_{ds} = \beta \left[(V_{gs} - V_t)V_{ds} - \frac{V_{ds}^2}{2} \right] \quad 0 < V_{gs} < V_{gs} - V_t$$

$$\approx \beta [V_{gs} - V_t] V_{ds} \quad \text{When } V_{ds} \ll V_{gs} - V_t$$

(3) Saturation region

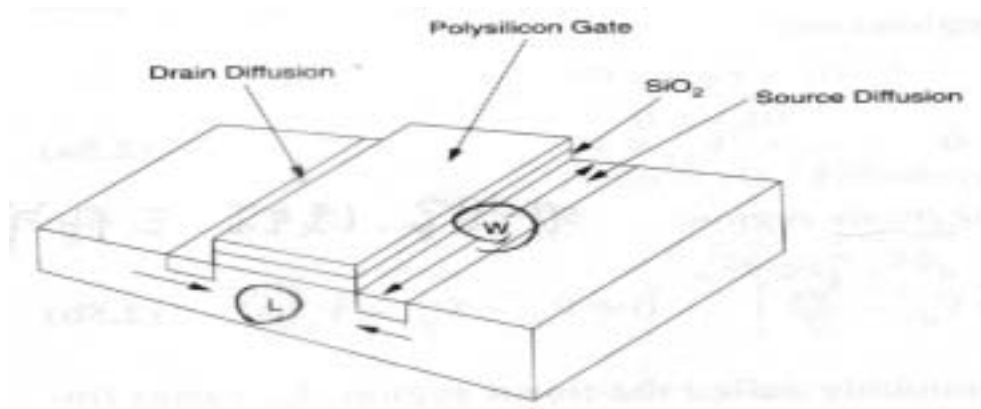
$$I_{ds} = \beta \frac{(V_{gs} - V_t)^2}{2}, \quad 0 < V_{gs} - V_t < V_{ds}$$



- V_d at which the device becomes saturated is called V_{dsat} (drain saturation voltage)

- β : MOS transistor gain factor

Function of (1) process parameter (2) device geometry



(1) μ = effective mobility of the carrier in the channel

(2) ϵ = permittivity of the gate oxide

(3) t_{ox} = thickness of the gate oxide

$$\text{Note: } \frac{\epsilon}{t_{ox}} = C_{ox} \Rightarrow \beta = \mu C_{ox} \left(\frac{W}{L} \right)$$

- Example

Typical CMOS

(~1 μ) process

(1) $\mu_n = 500 \text{ cm}^2/\text{V-sec}$

(2) $\epsilon_0 = 3.9 \epsilon_0 = 3.9 * 8.85 * 10^{-14} \text{ F/cm}$ (permittivity of SiO_2)

(3) $t_{ox} = 200 \text{ \AA}$

$$\beta_n = \frac{\mu_n \epsilon}{t_{ox}} \left(\frac{W}{L} \right) = 88.5 \frac{W}{L} \mu\text{A}/\text{V}^2$$

$$\mu_p = 180 \text{ cm}^2/\text{V-sec} \Rightarrow \beta_p = 31.9 \frac{W}{L} \mu\text{A}/\text{V}^2$$

$$\frac{\beta_n}{\beta_p} = 2.8 \quad (2 \sim 3 \text{ depending on process})$$

2.2.2. Seven Second-order Effect

- SPICE : Simulation Program with Integrated Circuit Emphasis
- LEVEL: 1,2,3
 - (1) Basic DC Equations + Some second-order effects
 - (2) Based on device physics
 - (3) Add more parameters to match real circuits

e.g., Process gain factor

SPICE : Kp (10-100 $\mu A/V^2$ with 10%-20% variation)

A. Channel-length modulation

- When an MOS device is in saturation.
- $L_{\text{eff}} = L - L_{\text{short}}$

$$L_{\text{short}} = \sqrt{2 \frac{\epsilon_{\text{si}}}{qN_A} (V_{\text{ds}} - (V_{\text{gs}} - V_t))}$$

$\Rightarrow L \Rightarrow \Rightarrow I_{\text{ds}}$

$$I_{\text{ds}} = \frac{K}{2} \left(\frac{W}{L} \right) (V_{\text{gs}} - V_t)^2 (1 + \lambda V_{\text{ds}})$$

With $K = \frac{\mu \epsilon}{t_{\text{ox}}}$: process gain factor

:channel length modulation factor ($0.02V^{-1}$ to $0.005 V^{-1}$)

(In SPICE level 1 : $\lambda = \text{LAMBDA}$)

B. Drain punchthrough (avalanche breakdown)

V_D is very high , I_{ds} is independent of V_{gs}
Good for I/O protection circuit.

C. Threshold voltage (Vt) – Body effect (Vsb)

- $$V_t = V_{fb} + 2\phi_b + \frac{\sqrt{2\varepsilon_{si}qN_A(2\phi_b + |V_{SB}|)}}{C_{ox}}$$

$$\Rightarrow V_t = V_{t0} + \gamma \left[\sqrt{2\phi_b + |V_{SB}|} - \sqrt{2\phi_b} \right]$$

(1) Vsb : Substrate bias

(2) Vt0 : Vt at Vsb=0

(3) γ : a constant which describes the substrate bias effect

(range:0.4~1.2)
$$\gamma = \frac{t_{ox}}{\varepsilon_{ox}} \sqrt{2q\varepsilon_{si}N_A} = \frac{1}{C_{ox}} \sqrt{2q\varepsilon_{si}N_A}$$

(4) SPICE

- γ : GAMMA in SPICE model
- Vto : VT0
- N_A : NSUB
- $\phi_b = 2\phi_b$: PHI (the surface potential at the onset of strong inversion)

Subthreshold region

- Cut-off = subthreshold region
- $I_{ds} \approx 0$ (Subthreshold region)
- But the finite value of I_{ds} may be used to construct very low power circuits.
- In Level 1 SPICE , subthreshold current is set 0

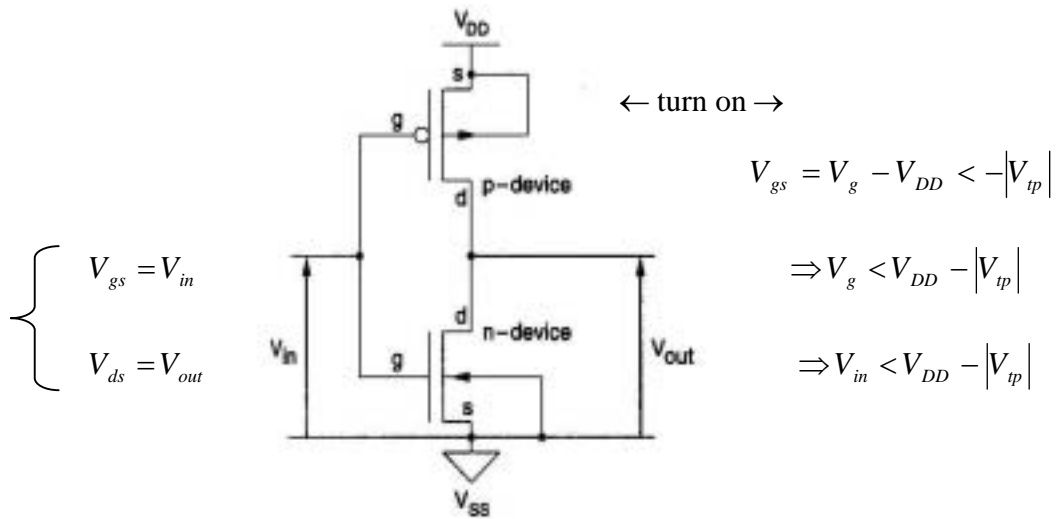
Others:

- **Mobility variation**
- **Fowler-Nordheim Tunneling**
- **Impact Ionization (Hot electrons effect)**

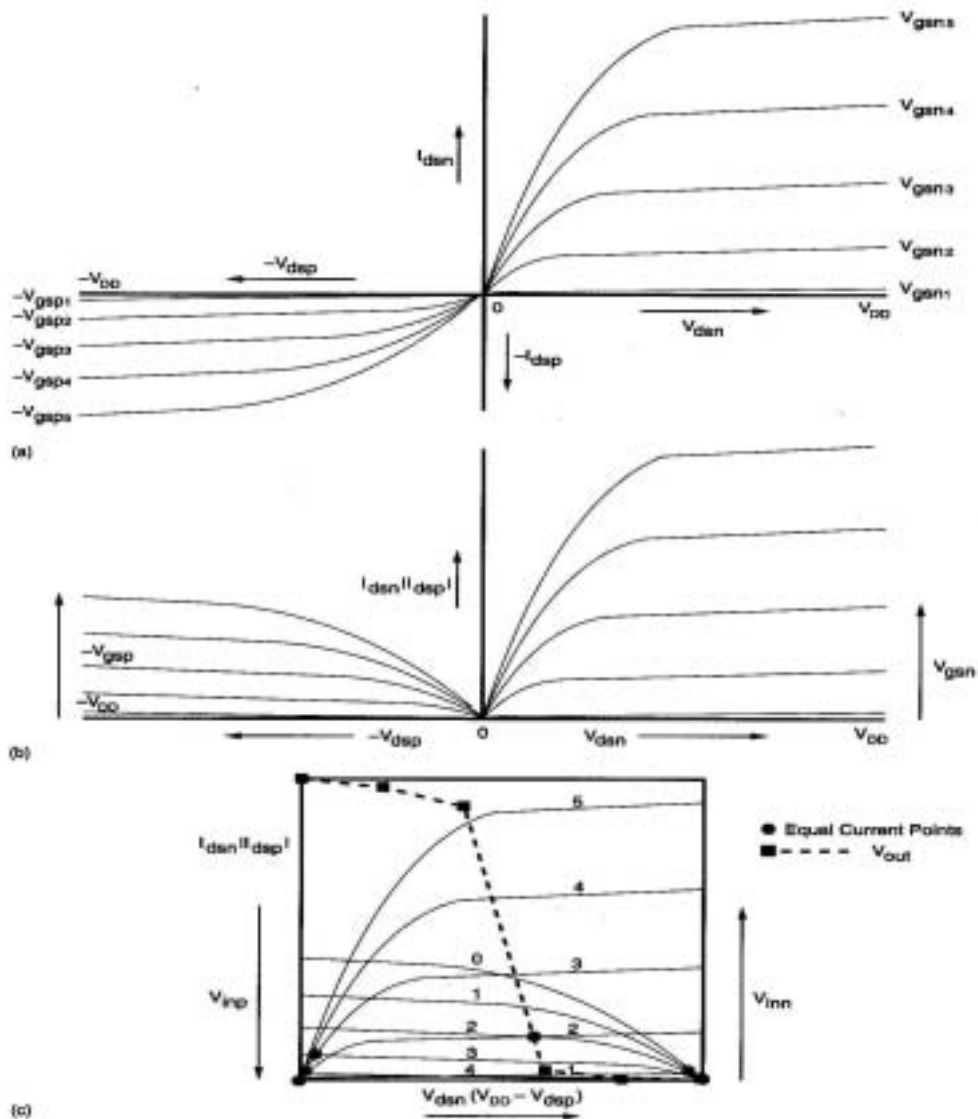
2.2.3 MOS Models

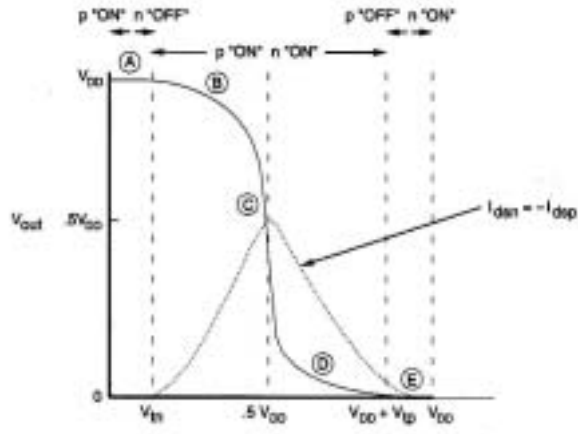
- MOS model = Ideal Equations + Second-order Effects + Additional Curve-fitting parameters
- Many semiconductor vendors expend a lot of effects to model the devices they manufacture.(Standard : Level 3 SPICE)
- Main SPICE DC parameters in level 1,2,3 in 1 μ n-well CMOS process.

2.3 CMOS inverter DC characteristics



(check Fig. 2.12)





Both transistors are "on"

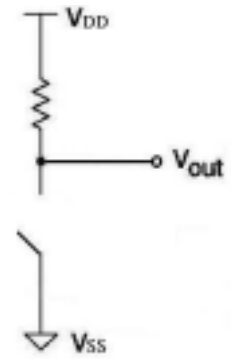
$$P = fcv^2 \cdot \alpha$$

↑ (Switching activity)

- Solve for $\begin{cases} I_{dsn} = -I_{dsp} \\ V_{inn} = V_{inp} \end{cases}$

(1) Region A. $0 \leq V_{in} \leq V_{tn}$

$$\begin{cases} \text{n-device is 'off', } I_{dsn} = 0 (= -I_{dsp}) \\ \text{p-device is in 'linear' mode} \\ V_{out} - V_{DD} = V_{dsp} = 0 \\ \Rightarrow V_{out} = V_{DD} \end{cases}$$

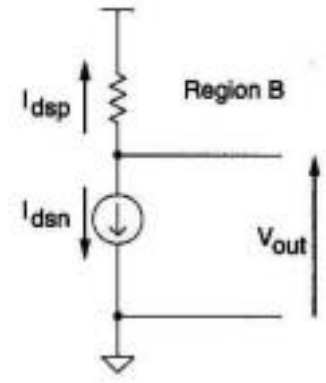
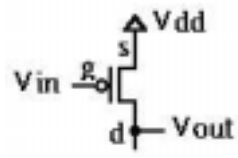


(2) Region B. $V_{tn} \leq V_{in} \leq V_{DD}/2$

$$\begin{cases} \text{p-device : linear mode} \\ \text{n-device : saturation mode} \end{cases}$$

$$n : I_{dsn} = \beta_n \frac{[V_{in} - V_{tn}]^2}{2}, \quad \beta_n = \frac{\mu_n \epsilon}{t_{ox}} \left(\frac{W_n}{L_n} \right)$$

$$p : \begin{cases} V_{gs} = V_{in} - V_{DD} \\ V_{ds} = V_{out} - V_{DD} \end{cases}$$



$$I_{dsp} = -\beta_p \left[\frac{(V_{in} - V_{DD} - V_{tp})(V_{out} - V_{DD})}{V_{gs} - V_{tp}} - \frac{(V_{out} - V_{DD})^2}{2} \right] V_{ds}$$

with $\beta_p = \frac{\mu_p \epsilon}{t_{ox}} \left(\frac{W_p}{L_p} \right)$

solve for $I_{dsp} = -I_{dsn}$

$$\Rightarrow V_{out} = (V_{in} - V_{tp}) + \sqrt{(V_{in} - V_{tp})^2 - 2(V_{in} - \frac{V_{DD}}{2} - V_{tp})V_{DD} - \frac{\beta_n}{\beta_p}(V_{in} - V_{tn})^2}$$

(3) Region C. PMOS, NMOS : saturation

$$I_{dsp} = -\frac{\beta_p}{2}(V_{in} - V_{DD} - V_{tp})^2$$

$$I_{dsn} = \frac{\beta_n}{2}(V_{in} - V_{tn})^2$$

with $I_{dsp} = -I_{dsn}$

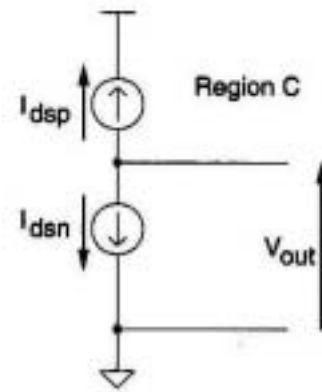
$$\Rightarrow V_{in} = \frac{V_{DD} + V_{tp} + V_{tn} \sqrt{\frac{\beta_n}{\beta_p}}}{1 + \sqrt{\frac{\beta_n}{\beta_p}}}$$

\Rightarrow by setting $\beta_n = \beta_p$ and $V_{tn} = -V_{tp}$

we have $\boxed{V_{in} = \frac{V_{DD}}{2}}$: one value only

possible V_{out}

$$\text{N-MOS} \begin{cases} V_{in} - V_{tn} < V_{out} \\ V_{gs} - V_{tn} < V_{ds} \end{cases}$$



$$\text{P-MOS} \begin{cases} (V_{DD} - V_{in}) - V_{tp} > (V_{DD} - V_{out}) \\ V_{gs} - V_{tp} > V_{ds} \end{cases}$$

$$\Rightarrow V_{out} < V_{in} - V_{tp}$$

$$\Rightarrow \boxed{V_{in} - V_{in} < V_{out} < V_{in} - V_{tp}}$$

Negative value

$$\Rightarrow V_{in} \text{ is fixed at } \frac{V_{DD}}{2}, V_{out} \text{ varies}$$

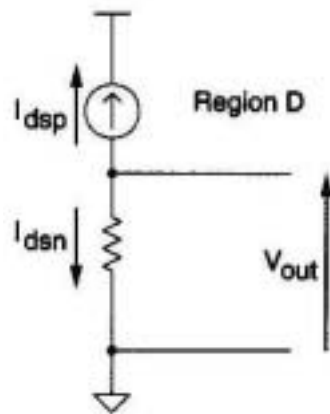
\Rightarrow make the o/p transition very steep

(4) Region D. $\frac{V_{DD}}{2} < V_{in} < V_{DD} + V_{tp}$

$\begin{cases} \text{P-MOS : saturation mode} \\ \text{N-MOS : linear mode} \end{cases}$

$$I_{dsp} = -\frac{1}{2} \beta_p (V_{in} - V_{DD} - V_{tp})^2$$

$$I_{dsn} = \beta_n \left[(V_{in} - V_{tn}) V_{out} - \frac{V_{out}^2}{2} \right]$$



solve $I_{dsn} = -I_{dsp}$

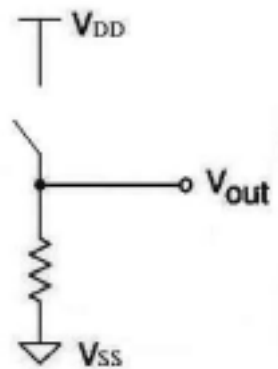
$$\Rightarrow V_{out} = (V_{in} - V_{tn}) - \sqrt{(V_{in} - V_{tn})^2 - \frac{\beta_p}{\beta_n} (V_{in} - V_{DD} - V_{tp})^2}$$

(5) Region E. $V_{in} \geq V_{DD} + V_{tp}$

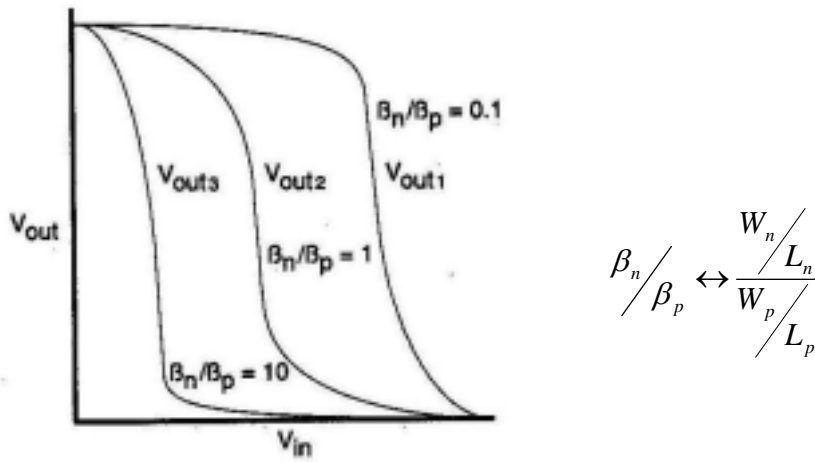
\rightarrow p-device ' off ' (n-device is in ' linear ' mode)

$$\rightarrow I_{dsp} = 0 \Rightarrow I_{dsn} = 0 \Rightarrow V_{out} = 0$$

see Table 2.3 for summary



2.3.1 n/p ratio (watch Eq.(2.24))

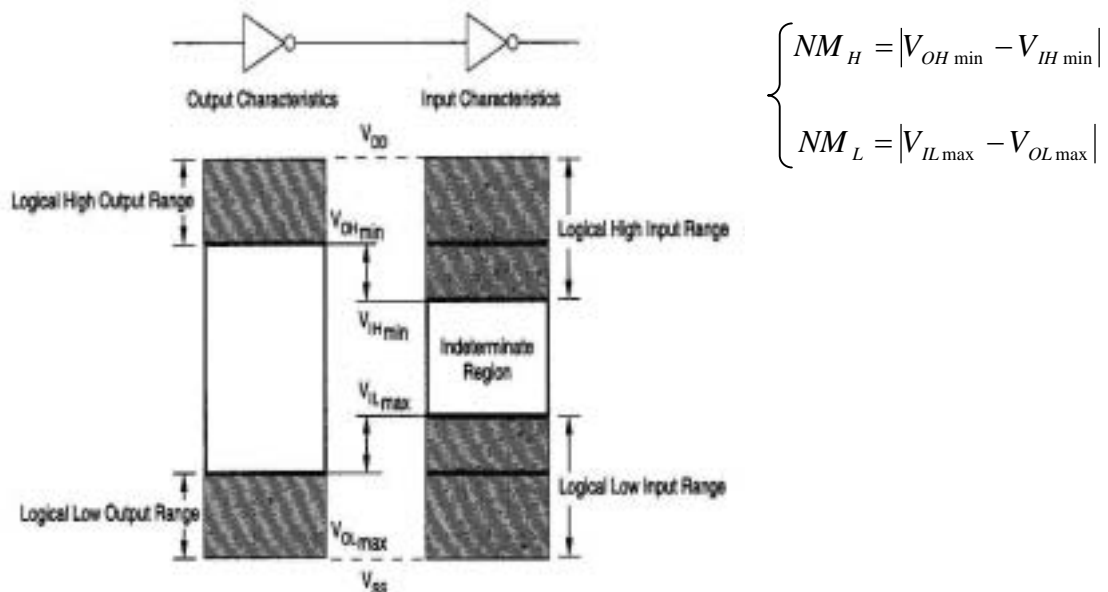


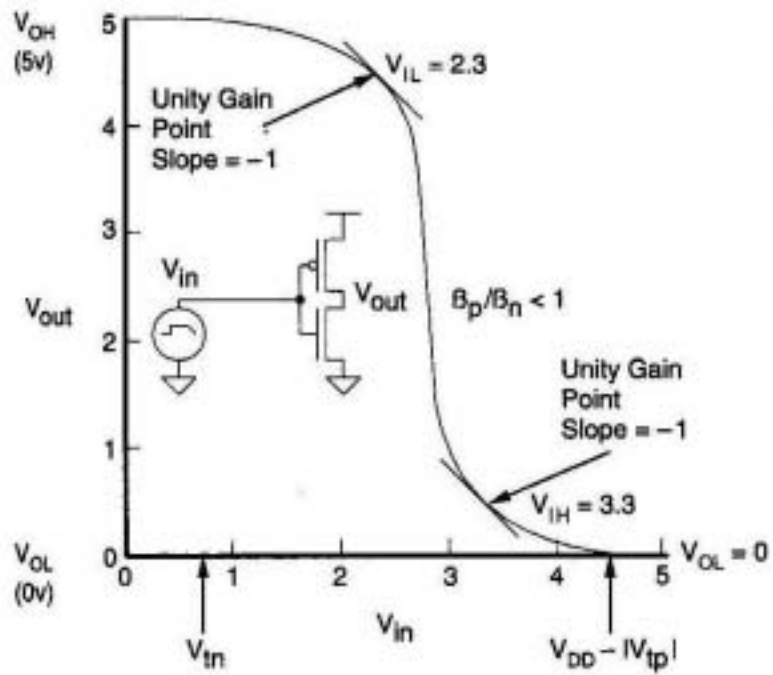
- Note $\beta \propto T^{-1.5}$ (T , μ)

$$\Rightarrow I_{ds} \propto T^{-1.5}$$

2.3.2 Noise Margin

- This parameter allows us to determine the allowable noise voltage on the input of a gate so that the output will not be affected.





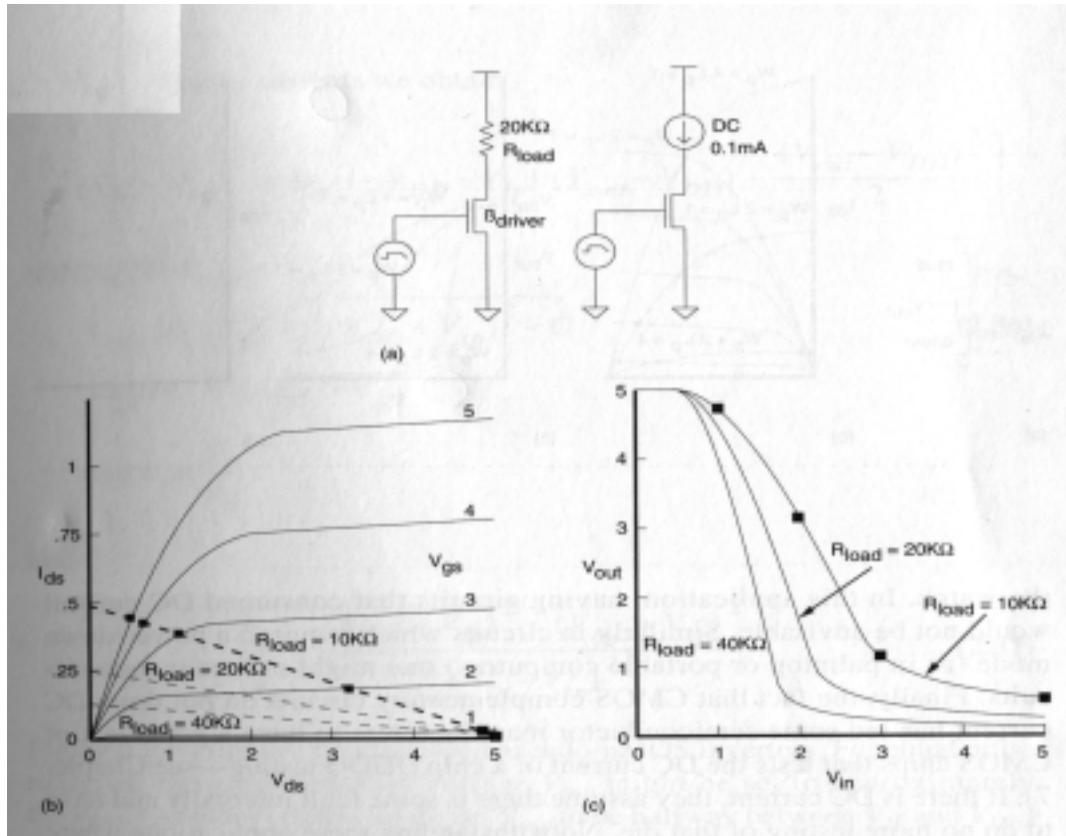
$$\begin{cases} V_{IL} = 2.3 \\ V_{IH} = 3.3 \end{cases}$$

$\begin{cases} V_{OL} \\ V_{OH} \end{cases}$ are more difficult (will be discussed later)

(left as your exercise)

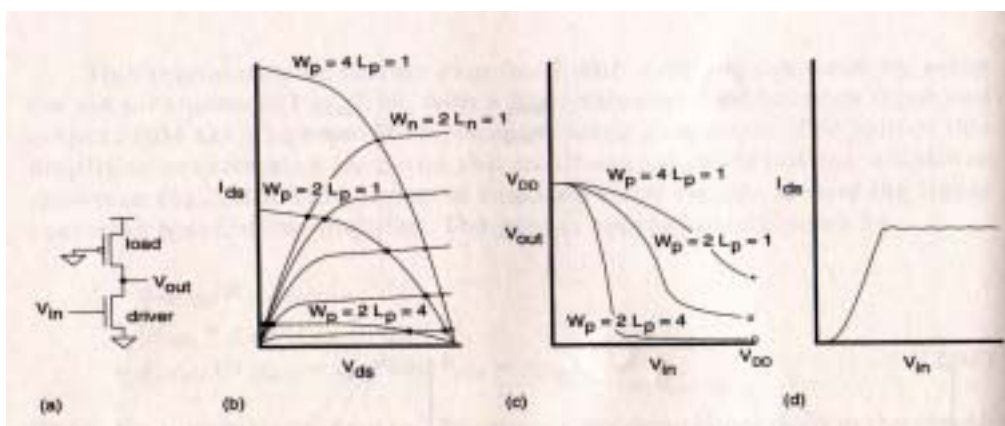
2.4 Static Load MOS inverters

- { Resistor-load inverter
Current-source-load inverter

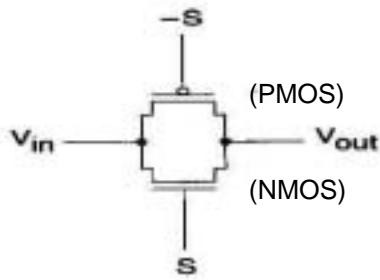


2.4.1 Pseudo-NMOS inverter

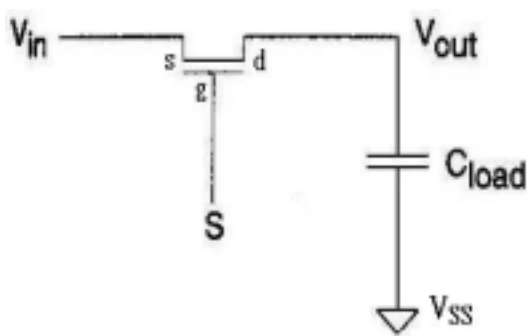
- Fast (constant current)
- power-consuming $P \Rightarrow I \Rightarrow$ but speed



2.6 The Transmission Gate



- NMOS pass transistor



C_{load} is initially discharged

$$V_{out} = V_{SS}$$

with $S=0$ (V_{SS})

$$V_{gs} = 0V$$

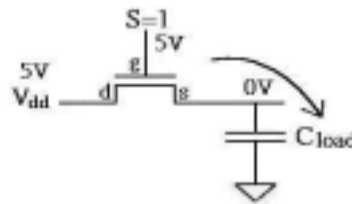
$$I_{ds} = 0$$

V_{out} remains at V_{SS}

$$V_{gs} = 5V > V_t, \text{ NMOS On} \Rightarrow V_{in} = V_{out} = 0V$$

$$S=1 \quad (V_{dd})$$

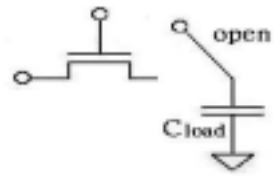
$$V_{gs} = V_{DD} \quad (\text{initially})$$



$$V_{gs} = 5V > V_t \Rightarrow \text{charge}$$

- $V_{in} > V_{out}$, current from V_{in} to V_{out}

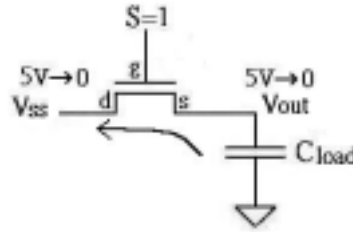
- As the output voltage approaches $V_{DD} - V_{in}$, the n-device begins to turn off



- $S=0$ (open circuit) , V_{out} remains at $V_{DD} - V_m(V_{dd})$,

where $V_m(V_{dd})$ denotes the V_t at $V_s = V_{dd}$ '(body effect)'

$$\begin{cases} S=1 \\ V_{in} = 0 \\ V_{out} = V_{DD} - V_m(V_{dd}) \end{cases}$$



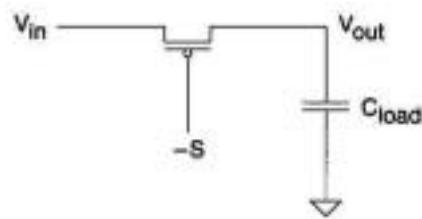
n-device begin to conduct , and V_{out} fall to V_{ss}

⇒ $\left\{ \begin{array}{l} \text{Transmission of Logic 1 is degraded, } (V_{DD} - V_m) \\ \text{Transmission of Logic 0 is not degraded, } (V_{ss}) \end{array} \right.$

- PMOS pass transistor

- $S=1$,(S=0) (open)

$$\begin{aligned} V_{in} &= V_{ss} \\ V_{out} &= V_{ss} \end{aligned}$$

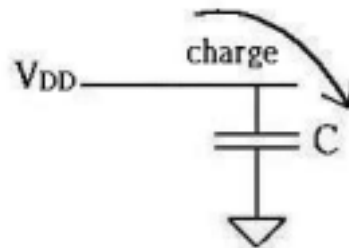


- $S=0$ (close)

$V_{in} = V_{DD}$, current

V_{out} to V_{DD}

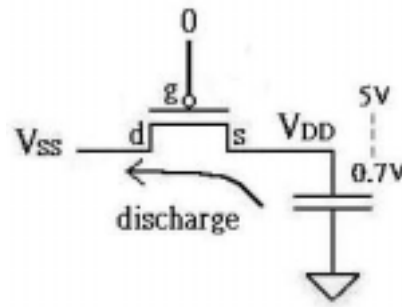
charge (C_{load})



- S=0 (close)

$$V_{in} = V_{SS} ,$$

$$V_{out} = V_{DD}$$



- Discharge C_load until through p-device

until $V_{out} = V_{tp}(V_{SS})$, at which point the transistor stops

conducting

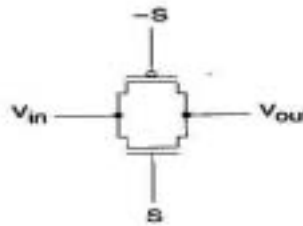
⇒ p-MOS passes good '1'
p-MOS passes poor '0'

- Transmission gate can pass logic '1' and '0' without degradation !

- overall behavior

- (1) S=0 ($\bar{S}=1$) :

$$\left\{ \begin{array}{l} \text{N,P devices are 'OFF'} \\ V_{in} = V_{SS} , V_{out} = Z \text{ (high impedance)} \\ V_{in} = V_{DD} , V_{out} = Z \end{array} \right.$$



- (2) S=1 ($\bar{S}=0$) :

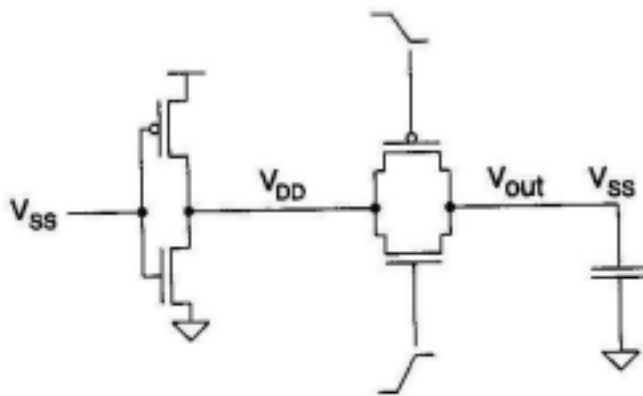
$$\left\{ \begin{array}{l} \text{N,P devices are 'ON'} \\ V_{in} = V_{SS} , V_{out} = V_{SS} , V_{in} = V_{DD} , V_{out} = V_{DD} \end{array} \right.$$

- Used in multiplexing element & latch element act as voltage-controlled resistor connecting the input and output

- Example to analyze a CMOS circuit

(Way 1=MAN
Way 2=SPICE)

(1) Capacitor loaded circuit



Clload at Vss

* Clload is large

- When S is ON $\left(\begin{array}{l} \text{NMOS, } S = \underline{0 \quad 1} \\ \text{PMOS, } \bar{S} = \underline{1 \quad 0} \end{array} \right)$

- Results: Currents of the pass transistor are monitored

V_{out} (transmission gate), $V_{gs}(p) = -5$ (constant current)

(PMOS)

It starts at 'saturation' \longrightarrow 'nonsaturation'

$$\text{as } |V_{gsp} - V_{tp}| > |V_{dsp}|$$

V_{out} (transmission gate)

(NMOS)

always at 'saturation', $V_{dsn} = V_{gsn}$

$$\Rightarrow V_{gsn} - V_{tn} < V_{dsn}$$

$\left\{ \begin{array}{l} \text{Rise} \\ \text{After } V_{out} \rightarrow V_{DD} - V_{tn}, \text{ NMOS is 'off'} \end{array} \right.$

■ Three regions of operation :

| | | | |
|---|--------------|-----------------|--|
| A | N saturation | P saturation | $V_{out} < V_{tp} $ |
| B | N saturation | P nonsaturation | $ V_{tp} < V_{out} < V_{DD} - V_{tn}$ |
| C | N off | P nonsaturation | $V_{DD} - V_{tn} < V_{out}$ |

a. Region A

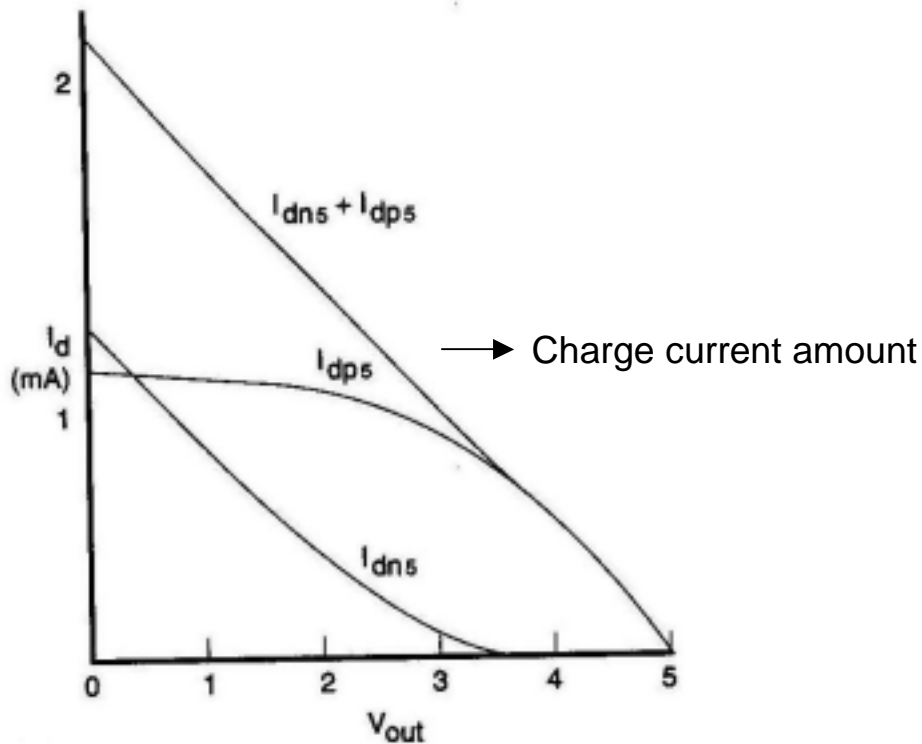
- { p : constant current source
- { n : current varies inversely with Vout

b. Region B

both currents vary linearly (inverse) with Vout

c. Region C

p-current varies inverse linearly with Vout



check : $V_{in} = V_{SS}, V_{out} = V_{DD} \rightarrow V_{SS}$

(2) Lightly loaded circuit (Cload is small)

■ Vout follows Vin very closely

■ Fig 2.35(d) $\left\{ \begin{array}{l} \text{n-current for } \frac{V_{out} - V_{in} = -0.1}{\dots} \\ \text{p-current} \end{array} \right.$

■ Three regions of operation :

- a. n (linear) , p (off)
- b. n (linear) , p (linear)
- c. n (off) , p (linear)

