chap 6 Karnaugh map

* Problems in algebraic simplification

1. The procedures are difficult to apply in a systematic way.
2. It is difficult to tell when you have arrived at a minimum solution. (minimum sop, pos)

⇒ Karnaugh map (K-map) is the solution

\[ \begin{array}{c|cc}
M_0 & A'B & AB \\
--- & --- & --- \\
0 & 0 & 0 \\
1 & 0 & 1 \\
\end{array} \]

\[ \begin{array}{c|cc}
M_1 & A'B & AB \\
--- & --- & --- \\
2 & 1 & 0 \\
3 & 1 & 0 \\
\end{array} \]

\[ A'B + AB = A' (B+0) = A' \]

\[ \begin{array}{c|ccc}
B'C & 00 & 01 & 11 \\
--- & --- & --- & --- \\
00 & 0 & 0 & 0 \\
01 & 0 & 1 & 0 \\
11 & 0 & 1 & 1 \\
10 & 0 & 0 & 1 \\
\end{array} \]
Ex. K-Map for \( F(a,b,c) = \Sigma m(1,3,5) \)

\[ = \Pi \bar{M}(0,2,4,6,7) \]

\[ F = \bar{A}C + B\bar{C} \]

\[ = \text{Minimum SOP form} \]

*Other combinations*

\[ F = B \]

\[ F = B\bar{C} + B\bar{C}' \]

\[ F = A\bar{C}' \]

\[ = C' \]

Ex1. \( f(a,b,c) = abc + bc + \bar{a} \)

\[ \Rightarrow F = \bar{A} + BC + BC' \]
Consensus theorem: $XZ + XZ + YZ = XZ + XZ$

$F = \Sigma m(0, 1, 2, 5, 6, 7)$

$F(a, b, c) = a'b' + bc' + ac$

$F(a, b, c) = a'c' + bc + ab$

6.3 4-variable of K-map

$F_{ac}, K$-map of $F(a, b, c, d)$

$= \text{ac} \text{d} + \text{a'} \text{b} + \text{d'} \text{c}$

Diagram of K-map with variables $A, B, C, D$.
Ex. 1. \( F = \Sigma m(1, 3, 4, 5, 10, 12, 13) \)

\[ F = BC' + AB'D + AB'CP' \]

* Circle of \( 2^k \)

\( \Rightarrow \) Eliminate \( K \) Variables

Ex. 2. \( F = \Sigma m(0, 2, 3, 5, 6, 7, 8, 10, 11, 14, 15) \)

\[ F = C + b'd' + a'bd \]

Ex. 3. \( F = \Sigma m(1, 3, 5, 7, 9) + \Sigma d(6, 12, 13) \)

\[ F = A'D + C'D \quad (SOP) \]

Ex. 4. Get POS form of minimum \( f(a, b, c, d) \)

\( \Rightarrow \) check SOP form of \( f'(a, b, c, d) \)

\( \Rightarrow \) Looping the "0"'s or a map of \( f(a, b, c, d) \)
6.4. Essential Prime Implicants.

* Implicant: Any single 1 or any group of 1's in the K-map of F function.

* Prime Implicant: If it cannot be combined with another term to eliminate a variable.

- A single 1 on a map represents a PI if it is not adjacent to any other 1's.
- Two adjacent 1's on a map form a PI if they are not contained in a group of 4 1's.
- and so on.
* Implicant: A product term (11個)
  
  - 5 minterms: \{AB'C, A'BC', A'B'C, ABC', ABC\}
  - 5 group of 2 minterms: \{A'B, AB, A'C, BC', BC\}
  - 1 group of 4 minterms: \{B\}

* Prime Implicant (PI): An implicant that is not covered by another implicant: \{B, A'C\}

* Essential Prime Implicant: A PI that covers at least one minterm that is not covered by another PI. \{B, A'C\} are also Essential PI.

⇒ An essential PI in the K-map by noting that it covers at least one minterm that is circled only once.

* Cover: A set of PI's which covers all 1's (all minterms)
All PI's:
\[ a'b'd, b'c', ac, a'c'd, ab, b'c'd \]
Essential PI = \{ b'c, ac \}

All PI's:
\[ \{ cd, bd, b'c, ac \} \]
Essential PI's:
\[ \{ bd, b'c, ac \} \]
\[ f = bd + b'c + ac \]

Essential PI's:
\[ \{ ac, a'b'd', ac'd \} \]
Two PI's can cover:
\[ A'b'cd \]
\[ \{ A'b'd, b'cd \} \]
\[ f = ac + a'b'd' + ac'd + \{ a'b'd \} \]
\[ \text{or} \ b'cd \]
* Rule: ① Find all Essential PI's

(Fig. 6-19) ② Find a minimum set of PI's to cover the remaining 1's on the map.

chap 7: Quine-McClusky Method

⇒ Computer algorithm to perform logic minimization (略)

Exam: Try

\[ f(A, B, C, D) = B'D' + B'C' + BCD \]