WMMSE-Based Alternating Optimization for Low-Complexity Multi-IRS MIMO Communication

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***Abstract*—Recently, intelligent reflecting surface (IRS) has emerged as a promising cost-efficient technology to enhance communication performance. In this paper, we start with a single-user IRS-assisted multiple-input-multiple-output (MIMO) system, aiming to maximize spectral efficiency. To deal with the complicated non-convex problem, we exploit the equivalence between the weighted mean square error minimization (WMMSE) and the spectral efficiency maximization problem, and propose a low-complexity and low-latency WMMSE-based alternating optimization (WMMSE-AO). In addition, it can be executed in parallel for further speedup of the IRS computation due to its non-coupling characteristic. Moreover, to fully exploit spatial multiplexing, we extend our proposed WMMSE-AO to general multi-IRS systems with better performance and coverage. Simulation results show that the proposed WMMSE-AO can reduce 20 times multiplications with only 1% performance degradation compared with the state-of-the-art algorithm.**

***Index Terms*—Alternating optimization, MIMO, multi-intelligent reflecting surface (IRS), weighted mean square error minimization (WMMSE).**

I. Introduction

To fulfill the explosive demand of data transmission in both beyond fifth-generation (B5G) and sixth-generation (6G) communication, intelligent reflecting surface (IRS) has recently emerged as a promising cost-efficient technology to enhance the communication performance by reflecting the incident signal to reconfigure the propagation environment. Without containing any active radio frequency (RF) chains to re-transmit the signals toward the destination, the low-cost IRS only passively reflects the signals by dynamically adjusting amplitude and phase, namely increasing the degree-of-freedom (DoF) in the spatial domain. Besides, IRS can be easily deployed and removed around the environments [1]. Motivated by the above advantages, there are plenty of studies on the joint design of the IRS coefficients and the active precoding matrix at the base station (BS) to optimize the system performance [2]–[4]. However, most works focused on multiple-input-single-output (MISO) systems because the objective of spectral efficiency (SE) under these scenarios is much easier to analyze compared with MIMO systems. To fully exploit the spatial multiplexing for the performance enhancement, we focus on the scenarios of MIMO systems as [2]–[4]. However, most of them suffer from high complexity in general.

Moreover, due to the double path loss effect, the performance enhancement can benefit from IRS beamforming gain only when the IRS is close to user equipment (UE) or base station (BS) [1]. As a result, the coverage is still limited even though the peak performance is high. To address the drawback of short-range coverage, we further extend our work to multiple IRSs [5]–[8]. The research on multiple IRSs can be divided into two scenarios. Scenario1 is the multiple IRS-assisted cooperative system with the inter-IRS channels considered to create a virtual LoS in the poor environment [5], [6]. Scenario2 is the multiple IRS-assisted system without the inter-IRS channels, mainly focusing on the coverage issue [7], [8]. Although multiple IRSs work efficiently in both two scenarios to improve performance, there is a lack of general algorithms that can be directly applied in both scenarios.

In this paper, we aim to maximize the spectral efficiency via the optimization of the equivalent weighted mean square error minimization (WMMSE) problem inspired by [9]. The new surrogate WMMSE objective avoids the determinant operation, thus providing the promise for an efficient optimization algorithm. Therefore, our proposed WMMSE-based alternating optimization (WMMSE-AO) can achieve high performance with low complexity and low latency. Moreover, our proposed WMMSE-AO can be extended to the general Multi-WMMSE-AO where the optimization considers every path including the inter-IRS channels. Therefore, Multi-WMMSE-AO can be easily adapted to the general environments whether the inter-IRS channels are considered. The main contributions of this paper are summarized below:

1) *Efficient WMMSE-based objective for single IRS*: We exploit the equivalence between the spectral efficiency maximization problem and the WMMSE problem whose objective avoids the complicated determinant operation. Therefore, the proposed WMMSE-AO can achieve 20 times complex multiplications reduction with 1% performance degradation compared with the state-of-the-art algorithm [4]. Moreover, the closed-form non-coupling IRS solution enables parallel optimization, thus achieving lower latency.
2) *Generalized Multi-WMMSE-AO for multi-IRS*: To further enhance the performance and coverage, the proposed WMMSE-AO can be extended to the general multi-IRS environments even the inter-IRS channels are considered. In particular, the IRSs can be parallelly updated in Scenario2 where the inter-IRS channels are neglected. Therefore, the extremely low latency property is suitable for practical applications.

II. System Model and Prior Works

A. System Model

Without loss of generality, we first consider a downlink single-user MIMO system with N_t antennas at the BS and N_r antennas at the UE, and the single IRS equipped with M passive reflecting elements is deployed to enhance the communication performance. The BS sends N_s data streams to the UE, with $N_s \leq \min\{N_t, N_r\}$. In the transmission, the received signal $\hat{\boldsymbol{s}} \in \mathbb{C}^{N_s \times 1}$ at the UE can be written as

$$\hat{\boldsymbol{s}} = \mathbf{W}^H \tilde{\mathbf{H}} \mathbf{F} \boldsymbol{s} + \mathbf{W}^H \boldsymbol{n}, \tag{1}$$

where $\tilde{\mathbf{H}} = \mathbf{H_r}^H \mathbf{\Theta} \mathbf{G} + \mathbf{H_d}$ is the effective channel. $\boldsymbol{s} \in \mathbb{C}^{N_s \times 1}$ is the transmitted signal vector with the assumption of $\mathrm{E}\left[\boldsymbol{s}\boldsymbol{s}^H\right] = \frac{P}{N_s}\mathbf{I}_{N_s}$, and P is the total transmit power. $\mathbf{F} \in \mathbb{C}^{N_t \times N_s}$ is the baseband digital precoder; $\mathbf{W} \in \mathbb{C}^{N_r \times N_s}$ is the baseband digital combiner, and the precoder is normalized as $\|\mathbf{F}\|_F^2 = N_s$ to satisfy the power constraint. Let $\boldsymbol{n} \in \mathbb{C}^{N_r \times 1} \sim \mathcal{CN}(\mathbf{0}, \sigma_n^2 \mathbf{I}_{N_r})$ denote the additive white Gaussian noise, where σ_n^2 is the noise variance.

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The effective channel $\tilde{\mathbf{H}}$ comprises of two links. One is the direct channel $\mathbf{H_d} \in \mathbb{C}^{N_r \times N_t}$ from the BS to the UE and the other one is the IRS-related channel composed of the BS-IRS channel $\mathbf{G} \in \mathbb{C}^{M \times N_t}$ and the UE-IRS channel $\mathbf{H_r} \in \mathbb{C}^{M \times N_r}$. The diagonal matrix $\mathbf{\Theta} \in \mathbb{C}^{M \times M}$ of the IRS is denoted by $\mathbf{\Theta} = \mathrm{diag}([\gamma_1 e^{j\theta_1}, \gamma_2 e^{j\theta_2}, \ldots, \gamma_M e^{j\theta_M}]^T)$, where $\gamma_m \in [0,1]$ and $\theta_m \in [0, 2\pi)$ denote the amplitude and the phase shift of the m-th reflecting element, respectively, $\forall m \in \mathcal{M} = \{12, \ldots, M\}$. Note that diag$(\cdot)$ has two modes corresponding to the vector input or the matrix input. diag $(\boldsymbol{a})$ denotes a diagonal matrix with the elements of $\boldsymbol{a}$ on its main diagonal, while diag$(\mathbf{A})$ is a vector composed of diagonal elements of $\mathbf{A}$. We refer to [2]–[4] by assuming $\gamma_m = 1, \forall m \in \mathcal{M}$ for practical hardware implementation, which means each element acts as a phase shifter.

B. Channel Model

In this paper, we assume the perfect channel state information (CSI) is available at the BS and the IRS controller.[1] We adopt the Rayleigh fading channel model for all channels, i.e., $\mathbf{H_d}$, $\mathbf{G}$ and $\mathbf{H_r}$. For generality, the Rayleigh fading channel model $\mathbf{H}$ can be expressed as

$$\mathbf{H} = \sqrt{\beta(d)}\, \mathbf{H^{NLoS}}, \tag{2}$$

where $\beta(d) = \beta_0 (d/d_0)^{-\alpha}$ is the distance-dependent path loss. β_0 is the path loss at the reference distance d_0, and α denotes the path loss exponent. $\mathbf{H^{NLoS}}$ is the normalized Rayleigh fading channel whose entries are randomly generated from the independent and identically distribution of $\mathcal{CN}(0,1)$.

C. Problem Formulation and Prior Works

The objective is to jointly optimize the precoder $\mathbf{F}$, the combiner $\mathbf{W}$ and the IRS $\mathbf{\Theta}$ to maximize spectral efficiency (SE) which can be written as

$$\mathrm{SE}(\mathbf{F}, \mathbf{W}, \mathbf{\Theta}) = \log\det\left(\mathbf{I}_{N_s} + \frac{P}{N_s}\mathbf{R}^{-1}\mathbf{W}^H\tilde{\mathbf{H}}\mathbf{F}\mathbf{F}^H\tilde{\mathbf{H}}^H\mathbf{W}\right), \tag{3}$$

where $\mathbf{R} = \sigma_n^2 \mathbf{W}^{\mathrm{H}}\mathbf{W}$ is the noise covariance matrix. Hence, spectral efficiency maximization problem can be formulated as

$$\begin{aligned} (\mathbf{P1}): \quad & \max_{\mathbf{F},\mathbf{W},\mathbf{\Theta}} \ \mathrm{SE}(\mathbf{F}, \mathbf{W}, \mathbf{\Theta}) \\ & \text{s.t.} \quad \|\mathbf{F}\|_F^2 = N_s \\ & \qquad \left|[\mathbf{\Theta}]_{i,i}\right| = 1, \ \forall i \in \mathcal{M}. \end{aligned}$$

However, due to the unit-modulus constraint in each IRS reflecting element, (P1) is a non-convex optimization problem.

To avoid the complicated analysis of determinant operation in the objective of spectral efficiency, [2], [3] simplified the optimization problems. In [2], the fixed-point iteration (FPI) method was adopted to optimize the IRS matrix, which relied on the estimated individual subchannels of each reflecting element. In [3], the sum-path-gain maximization (SPGM) objective can be solved via the alternating direction method of multipliers (ADMM) algorithm. Nevertheless, FPI and SPGM algorithms cannot achieve high performance and high scalability owing to the simplified optimization problems. On the other hand, [4] proposed an alternating optimization (AO) method in terms of spectral efficiency (SE-AO). By transferring the objective into several subproblems in terms of different reflecting elements, it alternatively optimizes one of the precoder and each reflecting element while fixing the others. In addition, a low-complexity algorithm (SE-LowComplex) was also proposed in [4] by omitting the consideration of power allocation in high effective signal-to-noise (SNR) environments. Although SE-AO can achieve high performance, it still has two critical issues:

1) *High complexity:* To directly optimize the spectral efficiency, it involves complicated operations such as matrix inverse and eigenvalue decomposition to decompose and optimize the problem.
2) *Long latency due to element-wise update:* The update of each reflecting element is limited to serial optimization due to the coupling effect. Therefore, IRS optimization will suffer from long latency.

Our goal is to develop a low-complexity algorithm for multi-IRS scenarios while avoiding these drawbacks of prior works.

III. Proposed WMMSE/MMSE-AO

In this section, to reduce the complexity and achieve parallel update of IRS reflecting elements, we aim to find a surrogate objective and derive its solution. Hence, we formulate the WMMSE problem and its equivalence to the original problem (P1). Based on the WMMSE objective, we propose a WMMSE-based alternating optimization (WMMSE-AO) to achieve high performance with lower complexity and latency.

A. Weighted Mean Square Error Minimization Problem

From [9], the surrogate performance metric MSE matrix $\mathbf{E} \in \mathbb{C}^{N_s \times N_s}$ can be expressed as

$$\begin{aligned} \mathbf{E} &= \mathrm{E}\left[(\hat{\boldsymbol{s}} - \boldsymbol{s})(\hat{\boldsymbol{s}} - \boldsymbol{s})^H\right] \\ &= \frac{P}{N_s}\left(\mathbf{I}_{N_s} - \mathbf{W}^H\tilde{\mathbf{H}}\mathbf{F}\right)\left(\mathbf{I}_{N_s} - \mathbf{W}^H\tilde{\mathbf{H}}\mathbf{F}\right)^H \\ &\quad + \sigma_n^2 \mathbf{W}^H\mathbf{W} \end{aligned} \tag{4}$$

Defining a positive semi-definite matrix $\mathbf{Z} \in \mathbb{C}^{N_s \times N_s} \succcurlyeq \mathbf{0}$ as the weight matrix, the WMMSE problem can be formulated as

$$\begin{aligned} (\mathbf{P2}): \quad & \min_{\mathbf{Z},\mathbf{F},\mathbf{W},\mathbf{\Theta}} \ \mathrm{tr}(\mathbf{ZE}) - \log\det(\mathbf{Z}) \\ & \text{s.t.} \quad \|\mathbf{F}\|_F^2 = N_s, \\ & \qquad \left|[\mathbf{\Theta}]_{i,i}\right| = 1, \ \forall i \in \mathcal{M}. \end{aligned} \tag{5}$$

Then, the following theorem establishes the equivalence between the SE maximization problem (P1) and the WMMSE problem (P2).

Theorem 1: The WMMSE problem (P2) is equivalent to the SE maximization problem (P1), namely the global optimal solutions $\mathbf{F}$, $\mathbf{\Theta}$ are identical for these two problems.

Please refer to [9] for detailed derivations and proof of the equivalent problem transformation.

B. Optimization Design

Although the problem (P2) avoids the determinant operation, it is still non-convex due to the unit-modulus constraint and the highly coupled variables. To handle such a highly intractable non-convex problem, the iterative AO-based method is proposed by decomposing (P2) into three sub-problems.

[1]Recently, many prior works have been dedicated to addressing the channel acquisition issues for IRS-assisted MIMO systems [2], [10], [11]. In [2], the authors proposed the orthogonal DFT or Hadamard matrix as the reflection pattern to estimate sub-channels based on least-square (LS) method. Moreover, [10] shows that the parallel factor (PARAFAC) model structure in received signals can be exploited, and thus proposed two novel methods to estimate the channels efficiently. As for the double IRS with inter-IRS channel, the tensor structure is utilized in [11].

1) *Precoder/Combiner* $\mathbf{F}, \mathbf{W}$ *optimization:* Given a fixed $\boldsymbol{\Theta}$ and $\mathbf{Z}$, the optimal combiner can be obtained via the MMSE combiner $\mathbf{W}^{MMSE} = \mathbf{J}^{-1}\ \tilde{\mathbf{H}}\mathbf{F}$, where $\mathbf{J}\ = \tilde{\mathbf{H}}\ \mathbf{F}\mathbf{F}^H\tilde{\mathbf{H}}^H + \frac{N_s\sigma_n^2}{P}\mathbf{I}_{N_r}$ is the covariance matrix with the perfect prior knowledge of the noise variance σ_n^2. However, in the practical time-varying channels, the noise variance is hard to be estimated and high-cost. Alternatively, in terms of SE, the optimal fully-digital precoder $\mathbf{F}^{svd}$ and combiner $\mathbf{W}^{svd}$ can be obtained via SVD of the effective channel $\tilde{\mathbf{H}}$ with power allocation as in [2]–[4]. Denote $\tilde{\mathbf{H}} = \tilde{\mathbf{U}}\ \tilde{\boldsymbol{\Sigma}}\tilde{\mathbf{V}}^H$as the truncated SVD of the effective channel, where $\tilde{\mathbf{U}} \in \mathbb{C}^{N_r\times N_s}$ is the left singular vectors, $\tilde{\mathbf{V}} \in \mathbb{C}^{N_t\times N_s}$ is the right singular vectors, $\tilde{\boldsymbol{\Sigma}} \in \mathbb{C}^{N_s\times N_s}$ is a diagonal matrix whose diagonal elements are the singular values in descending order. For simplicity and low-complexity concern, we apply the near-optimal solutions with equal power allocation due to the high SNR provided by the IRS beamforming gain [2]. Therefore, the near-optimal fully-digital precoder and combiner can be expressed as

$$\mathbf{F}^{svd}\ = \tilde{\mathbf{V}}\ (:, 1{:}N_s)\,, \ \mathbf{W}^{svd} = \tilde{\mathbf{U}}\ (:, 1{:}N_s)\,. \tag{6}$$

2) *IRS Matrix* $\boldsymbol{\Theta}$ *Optimization:* Given the fixed $\mathbf{F}, \mathbf{W}, \mathbf{Z}$ and neglecting the independent terms, (P2) can be expanded as

$$\begin{aligned}(\mathbf{P3}):\ &\max_{\boldsymbol{\Theta}}\ 2\mathrm{Re}\left\{\mathrm{tr}\left(\mathbf{Z}\hat{\mathbf{H}}\right)\right\} - \mathrm{tr}\left(\hat{\mathbf{H}}^H\mathbf{Z}\hat{\mathbf{H}}\right)\\ &\text{s.t.}\ \ \left|[\boldsymbol{\Theta}]_{i,i}\right| = 1,\ \forall i\in\mathcal{M},\end{aligned} \tag{7}$$

where $\hat{\mathbf{H}} = \mathbf{W}^H\ \tilde{\mathbf{H}}\mathbf{F}$. Then, (7) can be upper bounded by $2\mathrm{Re}\{\mathrm{tr}(\mathbf{Z}\hat{\mathbf{H}})\}$ because $\mathrm{tr}(\hat{\mathbf{H}}^H\mathbf{Z}\hat{\mathbf{H}})$ is always nonnegative. Note that the bound is tight when $\mathrm{tr}(\hat{\mathbf{H}}^H\mathbf{Z}\hat{\mathbf{H}}) \ll 2|\mathrm{Re}\{\mathrm{tr}(\mathbf{Z}\hat{\mathbf{H}})\}|$.

Due to the degraded channels, i.e., $\mathbf{H_d}, \mathbf{G}, \mathbf{H_r}$, caused by severe path loss $\beta(d)$, the power of the elements in $\hat{\mathbf{H}}$ are much smaller than the unit. Thus, following inequality can be claimed.

$$\mathrm{tr}\left(\hat{\mathbf{H}}^H\times\mathbf{Z}\hat{\mathbf{H}}\right) \ll \left|\mathrm{Re}\left\{\mathrm{tr}\left(\mathbf{Z}\hat{\mathbf{H}}\right)\right\}\right| < 2|\mathrm{Re}\left\{\mathrm{tr}\left(\mathbf{Z}\hat{\mathbf{H}}\right)\right\}|. \tag{8}$$

Therefore, we derive the tight upper bound of (7) as

$$2\mathrm{Re}\left\{\mathrm{tr}\left(\mathbf{Z}\mathbf{W}^H\mathbf{H}_{\boldsymbol{r}}^H\boldsymbol{\Theta}\mathbf{G}\mathbf{F}\right)\right\} + 2\mathrm{Re}\left\{\mathrm{tr}\left(\mathbf{Z}\mathbf{W}^H\mathbf{H_d}\mathbf{F}\right)\right\}. \tag{9}$$

By neglecting the constant second term in (9) and defining $\mathbf{H}'_{\mathbf{r}} = \mathbf{H_r}\mathbf{W}\mathbf{Z}^H \in \mathbb{C}^{M\times N_s}$, $\mathbf{G}' = \mathbf{G}\mathbf{F} \in \mathbb{C}^{M\times N_s}$, (P3) can be further simplified as the upper bound optimization problem as

$$\begin{aligned}(\mathbf{P4}):\ &\max_{\boldsymbol{\Theta}}\ \ \mathrm{Re}\left\{\mathrm{tr}\left(\mathbf{H}_{\mathbf{r}}'^H\boldsymbol{\Theta}\mathbf{G}'\right)\right\}\\ &\text{s.t.}\ \ \left|[\boldsymbol{\Theta}]_{i,i}\right| = 1,\ \forall i\in\mathcal{M}.\end{aligned} \tag{10}$$

We first decompose (P4) into several smaller subproblems (P4-m), namely the optimization of the m-th IRS reflecting element with the others being fixed. Denote $\mathbf{G}' = [\boldsymbol{g}_1', \boldsymbol{g}_2', \ldots, \boldsymbol{g}_M']^H$ and $\mathbf{H}'_{\mathbf{r}} = [\boldsymbol{h}_{\boldsymbol{r},1}', \boldsymbol{h}_{\boldsymbol{r},2}', \ldots, \boldsymbol{h}_{\boldsymbol{r},M}']^H$, where $\boldsymbol{g}_m'^H \in \mathbb{C}^{1\times N_s}$ is the m-th row of $\mathbf{G}'$ and $\boldsymbol{h}_{\boldsymbol{r},m}'^H \in \mathbb{C}^{1\times N_s}$ is the m-th row of $\mathbf{H}'_{\mathbf{r}}$. When the other reflecting elements are fixed, $\mathbf{H}_{\mathbf{r}}'^H\boldsymbol{\Theta}\mathbf{G}'$ can be represented as the form of summation

$$\begin{aligned}\mathbf{H}_{\mathbf{r}}'^H\boldsymbol{\Theta}\ \mathbf{G}' &= \sum_{i=1}^{M}\boldsymbol{\Theta}_{i,i}\ \boldsymbol{h}_{\boldsymbol{r},i}'\boldsymbol{g}_i'^H\\ &= \boldsymbol{\Theta}_{m,m}\ \boldsymbol{h}_{\boldsymbol{r},m}'\boldsymbol{g}_m'^H + \sum_{i\,=\,1, i\neq m}^{M}\boldsymbol{\Theta}_{i,i}\boldsymbol{h}_{\boldsymbol{r},i}'\boldsymbol{g}_i'^H.\end{aligned} \tag{11}$$

TABLE I
COMPARISON OF COMPUTATIONAL COMPLEXITY

Algorithm	Number of Complex Multiplications
FPI [2]	$N_tN_s^2(M+1)^2 + p_{max}(M+1)^2 +$ $N_tN_r^2(M+1) + f_{SVD}^{mul}(N_r, N_t)$
SPGM [3]	$MN_tN_r + N_r(M^2N_t + MN_t + M)$ $+p_{max}\left((M+1)^2 + 2(M+1)\right)$ $+5(M+1)^3 + f_{SVD}^{mul}(N_r, N_t)$
SE-AO [4]	$N_{itr}(M(4N_s^3 + 8N_s^2 + 6N_s) + N_tN_rN_s +$ $N_tN_s^2 + MN_tN_s + MN_rN_s$ $+M(N_tN_r + N_r) + f_{SVD}^{mul}(N_r, N_t))$
SE-LowComplex [4]	$(3N_{itr}+1)M(N_tN_r + N_r) + f_{SVD}^{mul}(N_r, N_t)$
WMMSE-AO	$N_{itr}(M(N_tN_r + N_s(N_t + N_r + 1) + N_r) +$ $N_rN_s^2 + f_{SVD}^{mul}(N_r, N_t) + N_tN_rN_s + 2N_s^2N_r +$ $N_s^3 + f_{inv}^{mul}(N_s))$
MMSE-AO	$N_{itr}(M(N_tN_r + N_s(N_t + N_r + 1) + N_r)$ $+N_rN_s^2 + f_{SVD}^{mul}(N_r, N_t))$

As a result, only the first term in (11) is dependent on the m-th IRS reflecting element. Therefore, (P4-m) can be expressed as

$$\begin{aligned}(\mathbf{P4-m}):\ &\max_{\boldsymbol{\Theta}_{m,m}}\ \ \mathrm{Re}\left\{\mathrm{tr}\left(\boldsymbol{\Theta}_{m,m}\boldsymbol{h}_{\boldsymbol{r},m}'\boldsymbol{g}_m'^H\right)\right\}\\ =\ &\max_{\boldsymbol{\Theta}_{m,m}}\ \ \mathrm{Re}\left\{\boldsymbol{\Theta}_{m,m}\boldsymbol{g}_m'^H\boldsymbol{h}_{\boldsymbol{r},m}'\right\}\\ &\text{s.t.}\ \ |\boldsymbol{\Theta}_{m,m}| = 1.\end{aligned} \tag{12}$$

(12) holds due to $\mathrm{tr}\left(\boldsymbol{b}\boldsymbol{a}^H\right) = \boldsymbol{a}^H\ \boldsymbol{b}$ for any vectors $\boldsymbol{a}\in\mathbb{C}^n$ and $\boldsymbol{b}\in\mathbb{C}^n$. From (12), we can obtain the closed-form solution $\boldsymbol{\Theta}_{m,m}^{opt} = e^{-j\angle(\boldsymbol{g}_m'^H\boldsymbol{h}_{\boldsymbol{r},m}')}$ which maximizes the projection of $\boldsymbol{\Theta}_{m,m}\boldsymbol{g}_m'^H\boldsymbol{h}_{\boldsymbol{r},m}'$ on the real axis. It can be noticed that the optimal m-th IRS reflecting element is non-coupling with the others. Thus, the parallel update of the whole reflecting elements is feasible and the optimal solution can be written in a compact form as follows

$$\boldsymbol{\Theta}^{opt} = \ \mathrm{diag}\left(e^{-j\angle\mathrm{diag}\left(\mathbf{G}\mathbf{F}\mathbf{Z}\mathbf{W}^H\mathbf{H}_{\boldsymbol{r}}^H\right)}\right). \tag{13}$$

3) *Weight Matrix* $\mathbf{Z}$ *Optimization:* Given a fixed $\mathbf{F}, \mathbf{W}, \boldsymbol{\Theta}$, (P2) is convex with respect to $\mathbf{Z}$ thus the closed-form optimal $\mathbf{Z}^{opt}$ can be written as

$$\mathbf{Z}^{opt} = \left(\mathbf{E}^{MMSE}\right)^{-1}. \tag{14}$$

The optimal weight matrix in each iteration establishes the equivalence between the SE maximization problem and the WMMSE problem, which makes these two problems have an identical gradient with respect to the precoder and IRS matrix. It is worth noting that if we fix the weight matrix as the identity matrix, (P2) becomes the MMSE problem. The corresponding solution called MMSE-AO also shows the efficiency and the lower-complexity in the simulation results.

C. Analysis of Computational Complexity

We evaluate the computational complexity in terms of the number of complex-valued multiplications. The complexity of the proposed WMMSE/MMSE-AO and other prior works are listed in Table I. The AO-based algorithms converge in N_{itr} iterations, where N_{itr} is usually small and not larger than 10. FPI and SPGM only process SVD once but use p_{max} iterations to meet the stopping criterion in the IRS matrix optimization, where p_{max} becomes larger with increased M. For precise evaluation, the complexity of SVD and matrix inverse are referred to [12] and [13]. $f_{SVD}^{mul}(m, n) = C_{SVD}\, mn\min(m, n)$ is the number of complex multiplications of an $m \times n$ matrix SVD, where $C_{SVD} = 10$. $f_{inv}^{mul}(n) = (n^5 - 3n^4 + 6n^3 - 3n^2 + 2n - 3)/3$ is the number of complex multiplications of an $n \times n$ matrix inversion through elementary row transformation. In an IRS-assisted MIMO system where $M \gg \{N_t, N_r, N_s\}$, the dominant complexity of the proposed WMMSE/MMSE-AO is linear with M.

IV. Proposed Multi-WMMSE-AO for Multi-IRS

Although IRS can improve the performance, the coverage is still limited because the spectral efficiency is only improved when the UE is close to the IRS. Therefore, we extend the proposed low-complexity WMMSE-AO to multiple IRSs, called Multi-WMMSE-AO. The proposed Multi-WMMSE-AO can be adapted to the general multiple IRSs environments whether the inter-IRS channels are considered. In other words, the proposed IRS optimization can fully exploit the spatial multiplexing gain to improve spectral efficiency and coverage.

A. Scenario1: Multi-IRS With Inter-IRS Channel

With the assistance of K multiple IRSs, the objective is to jointly optimize the precoder $\mathbf{F}$, the combiner $\mathbf{W}$ and the IRS $\mathbf{\Theta}^{(k)}$ with $M^{(k)}$ reflecting elements on each planar, $\forall k \in \mathcal{K} = \{1, 2, \ldots, K\}$, to maximize spectral efficiency. Assume the signal does not reflect over more than two IRSs due to the negligible power caused by the high path loss. The IRS-related channel is composed of the BS-IRS$^{(k)}$ channels $\mathbf{G}^{(k)} \in \mathbb{C}^{M^{(k)} \times N_t}$, the UE-IRS$^{(k)}$ channels $\mathbf{H}_{\mathbf{r}}^{(k)} \in \mathbb{C}^{M^{(k)} \times N_r}$, $\forall k \in \mathcal{K}$, and the inter-IRS channels between the i-th IRS and the j-th IRS $\mathbf{H}_{\mathbf{I}}^{(i,j)} \in \mathbb{C}^{M^{(j)} \times M^{(i)}}$, $\forall i, j \in \mathcal{K}$. Therefore, the optimization problem can be formulated in (P1), but the effective channel is defined as

$$\tilde{\mathbf{H}} = \mathbf{H}_{\mathbf{d}} + \sum_{i=1}^{K} \left(\left(\mathbf{H}_{\mathbf{r}}^{(i)H} + \sum_{j=1, j\neq i}^{K} \mathbf{H}_{\mathbf{r}}^{(j)H} \mathbf{\Theta}^{(j)} \mathbf{H}_{\mathbf{I}}^{(i,j)} \right) \mathbf{\Theta}^{(i)} \mathbf{G}^{(i)} \right). \tag{15}$$

The problem is hard to optimize due to the non-convexity, thus the WMMSE-based alternating optimization is exploited as the single IRS case. The detailed derivation is omitted because of the similarity as in Section III, and the problem is equivalent to

$$\begin{aligned} (\mathbf{P5}): \ & \max_{\{\mathbf{\Theta}^{(k)}\}} \ \mathrm{Re}\left\{ \mathrm{tr}\left(\mathbf{Z}\mathbf{W}^H \tilde{\mathbf{H}} \mathbf{F} \right) \right\} = \mathrm{Re}\left\{ \mathrm{tr}\left(\tilde{\mathbf{H}} \mathbf{F} \mathbf{Z} \mathbf{W}^H \right) \right\}, \\ & \text{s.t.} \quad \left| [\mathbf{\Theta}^{(k)}]_{i,i} \right| = 1, \forall k \in \mathcal{K}, \ \forall i \in \mathcal{M}^{(k)}, \end{aligned} \tag{16}$$

where $\mathcal{M}^{(k)} = \{1, 2, \ldots, M^{(k)}\}$, $\forall k \in \mathcal{K}$. By substituting (15) into (P5), it can be observed that the variables $\mathbf{\Theta}^{(k)}$, $\forall k \in \mathcal{K}$, are highly coupled with the others, which makes (P5) hard to optimize. To tackle such a problem, the AO-based optimization is utilized to optimize the k-th IRS with other IRSs being fixed. By neglecting the constant terms independent to $\mathbf{\Theta}^{(k)}$, the problem is equivalent as

$$\begin{aligned} (\mathbf{P5-k}): \ & \max_{\mathbf{\Theta}^{(k)}} \ \mathrm{Re}\left\{ \mathrm{tr}\left(\tilde{\mathbf{H}}^{(k)} \mathbf{F} \mathbf{Z} \mathbf{W}^H \right) \right\} \\ & \text{s.t.} \quad \left| [\mathbf{\Theta}^{(k)}]_{i,i} \right| = 1, \forall k \in \mathcal{K}, \ \forall i \in \mathcal{M}^{(k)}, \end{aligned} \tag{17}$$

where $\tilde{\mathbf{H}}^{(k)}$ is the summation of the multiple paths related to k-th IRS and formulated in (18), shown at the bottom of this page. The closed-form solution of (P5-k) is derived as (19), shown at the bottom of this page, based on derivation in Section III. The proposed Multi-WMMSE-AO can obtain the near-optimal solution under any environment owing to the generality to consider every channel path. It remains the advantages of fast convergence and low-complexity as in single WMMSE-AO.

B. Scenario2: Simplification Without Inter-IRS Channel

The proposed Multi-WMMSE-AO can be applied to any channel environment due to the general closed-form solution. However, the parallel update between the IRSs over the whole system is infeasible owing to the limitation of the coupling effect caused by the inter-IRS channels. Fortunately, in some special cases, the inter-IRS channel related paths are negligible [7], [8], namely $\mathbf{H}_{\mathbf{I}}^{(i,j)} = \mathbf{0}$, $\forall i, j \in \mathcal{K}$. The reason is that when all channels have similar channel conditions, the signal traveling over the BS-IRS-IRS-UE path will suffer from the triple path loss effect. Due to the severe power degradation, the signal of this path can be ignored compared with the BS-IRS-UE path which only suffers from the double path loss effect. In such special Scenario2, (19) can be simplified as

$$\mathbf{\Theta}_{opt}^{(k)} = \mathrm{diag}\left(e^{-j\angle \mathrm{diag}\left(\mathbf{G}^{(k)} \mathbf{F} \mathbf{Z} \mathbf{W}^H \mathbf{H}_{\mathbf{r}}^{(k)H} \right)} \right). \tag{20}$$

From (20), the update of the k-th IRS is non-coupling with the other IRSs. Therefore, the whole reflecting elements in the system can be updated at the same time to extremely shorten the latency of the multiple IRSs optimization.

V. Simulation Results

In this section, we present the numerical results of the proposed WMMSE/MMSE-AO and Multi-WMMSE-AO. The settings of the following system parameters are set as in [4]. Unless specified otherwise, we set the parameters as follows. The BS has $N_t = 4$ antennas and the UE has $N_r = 4$ antennas. The IRS planar comprises $M = 80$ reflecting elements. $N_s = 4$ data streams are considered. We set $d_0 = 1$ m and $\beta_0 = -30$ dB, and the path loss exponents of the BS-UE, BS-IRS, IRS-UE link are $\alpha_{\mathrm{H_d}} = 3.5$, $\alpha_{\mathrm{G}} = 2.2$, $\alpha_{\mathrm{H_r}} = 2.8$, respectively. We set P as 30 dBm and σ_n^2 as -90 dBm. For a fair comparison, N_{itr} for AO-based algorithms are all set as 10, and p_{max} in FPI and SPGM is variant until meeting the stopping criterion under different settings.

A. Proposed WMMSE/MMSE-AO Under Single IRS

In 3-dimensional xyz-plane, the BS, IRS, UE are located at (0,0,10), (170,2,10), (170,0,0) in meter (m), respectively.

$$\tilde{\mathbf{H}}^{(k)} = \mathbf{H}_{\mathbf{r}}^{(k)H} \mathbf{\Theta}^{(k)} \mathbf{G}^{(k)} + \sum_{i=1, i\neq k}^{k} \left(\mathbf{H}_{\mathbf{r}}^{(i)H} \mathbf{\Theta}^{(i)} \mathbf{H}_{\mathbf{I}}^{(k,i)} \mathbf{\Theta}^{(k)} \mathbf{G}^{(k)} + \mathbf{H}_{\mathbf{r}}^{(k)H} \mathbf{\Theta}^{(k)} \mathbf{H}_{\mathbf{I}}^{(i,k)} \mathbf{\Theta}^{(i)} \mathbf{G}^{(i)} \right). \tag{18}$$

$$\mathbf{\Theta}_{opt}^{(k)} = \mathrm{diag}\left(e^{-j\angle \mathrm{diag}\left(\mathbf{G}^{(k)} \mathbf{F} \mathbf{Z} \mathbf{W}^H \mathbf{H}_{\mathbf{r}}^{(k)H} + \sum_{i=1, i\neq k}^{K} \left(\mathbf{G}^{(k)} \mathbf{F} \mathbf{Z} \mathbf{W}^H \mathbf{H}_{\mathbf{r}}^{(i)H} \mathbf{\Theta}^{(i)} \mathbf{H}_{\mathbf{I}}^{(k,i)} + \mathbf{H}_{\mathbf{I}}^{(i,k)} \mathbf{\Theta}^{(i)} \mathbf{G}^{(i)} \mathbf{F} \mathbf{Z} \mathbf{W}^H \mathbf{H}_{\mathbf{r}}^{(k)H} \right) \right)} \right) \tag{19}$$

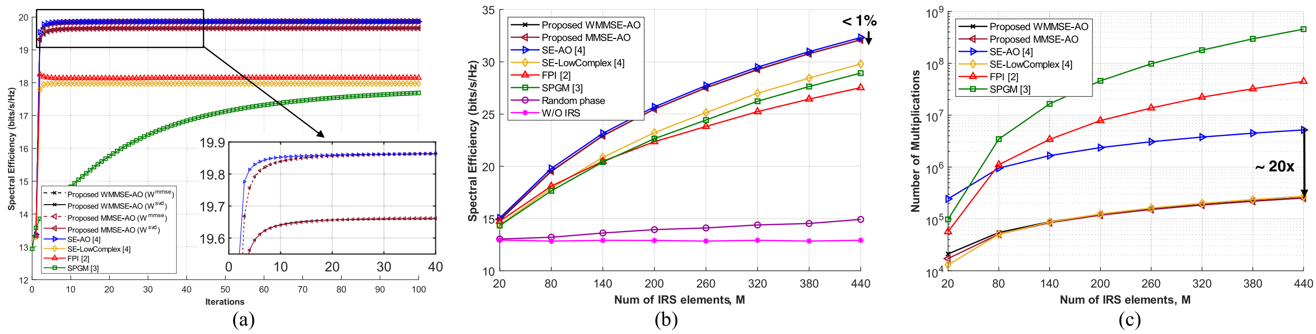

Fig. 1. (a) Convergence behavior. (b) Spectral efficiency versus M. (c) Computational complexity versus M.

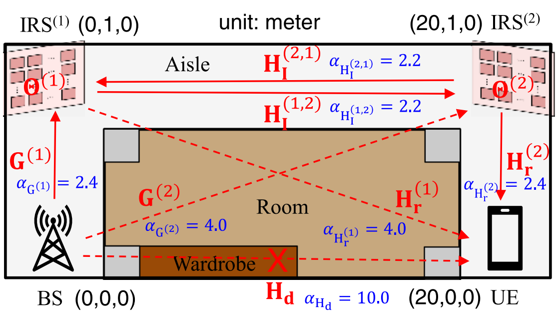

Fig. 2. Multiple IRS-assisted MIMO communication system.

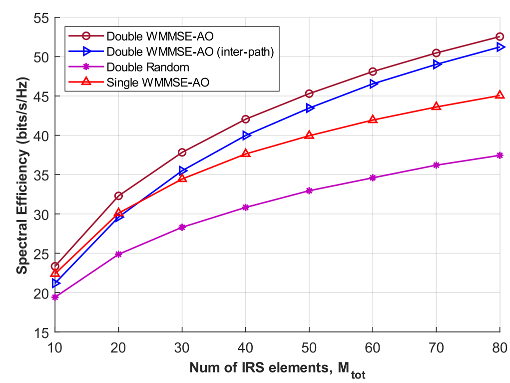

Fig. 3. Spectral efficiency versus M_{tot} under Scenario1.

Fig. 1(a) shows the fast convergence behavior of the algorithms. If the SNR is perfectly known as prior knowledge, the MMSE combiner based WMMSE/MMSE-AO can converge to the performance as the SE-AO, verifying the equivalent gradients of the SE maximization problem and the WMMSE problem. However, due to the lack of noise variance knowledge in practical applications, near-optimal combiner can be optimized based on SVD method, which also shows the high spectral efficiency compared to the others.[2] In the rest of this section, we use SVD-based combiner in WMMSE/MMSE-AO.

Fig. 1(b) shows the spectral efficiency versus M. When the M increases, our proposed WMMSE/MMSE-AO outperforms other algorithms and approaches SE-AO with the degradation less than 1%, which shows the scalability.

For the evaluation of scalability, we analyze the complexity based on Table I and plot the computational complexity in Fig. 1(c). Although SE-AO is also linear with M as proposed algorithms, WMMSE/MMSE-AO has less number of complex-valued multiplications. For the case of $M = 440$, our proposed WMMSE-AO can reduce 20 times number of multiplications while achieving comparable performance as SE-AO. Furthermore, due to the non-coupling property in (13), the proposed WMMSE/MMSE-AO can be calculated in parallel with M times reduction to achieve further lower latency.

B. Proposed Multi-WMMSE-AO in Scenario1

In the indoor Scenario1, shown in Fig. 2, the inter-IRS channels of the double IRSs and the different channel conditions caused by the obstacles such as walls and wardrobes are considered. The double IRSs equally distribute the M_{tot} total reflecting elements, i.e., $M_{tot}/2$ reflecting elements on each. The path loss exponents of channels in Fig. 2 are referred to [6]. In Fig. 3, we compare the spectral efficiency with three benchmarks: 1) **Double-WMMSE-AO (inter-path)** optimizes only on the inter-IRS channel related paths; 2) **Double-Random** randomly assigns each reflecting element phase of the double IRSs; 3) **Single-WMMSE-AO** allocates the total M_{tot} reflecting element on IRS$^{(2)}$. Several observations can be analyzed as follows. First, the curves of **Double-WMMSE-AO (inter-path)** and **Single-WMMSE-AO** exist an intersection, indicating that different schemes are suitable for different M_{tot}. When M_{tot} is small, the beamforming gain benefited from the double IRSs is not enough to compensate for the larger path loss over the longer transmission path. Thus, the single IRS with shorter BS-IRS$^{(2)}$-UE path can achieve higher performance. When M_{tot} is larger, the beamforming gain benefited from the double IRSs is higher to compensate the severer path loss and achieve higher performance. Moreover, the proposed **Double-WMMSE-AO** can always achieve the highest performance because the optimization considers every path in the environment, thus fully exploiting the spatial multiplexing gain.

In Scenario2, we further analyze the advantage of coverage for the multiple IRSs without inter-IRS channels. The BS is located at (0,0,10), and the UE moves along the x-axis from 80 to 250 meters with y-axis and z-axis at 0 meters to analyze the coverage in the cell. For a fair comparison, the following three schemes have the same number of total reflecting elements $M_{tot} = 240$. 1) **Single-WMMSE-AO** has only one IRS located at (170,2,10); 2) **Double-WMMSE-AO** has two IRSs located at (155,2,10) and (185,2,10); 3) **Triple-WMMSE-AO** has three IRSs located at (140,2,10), (170,2,10), and (200,2,10). To show the coverage of the multiple IRSs, we compare the three schemes in terms of the outage probability which is defined as

$$P_{out}(\tau) = P(\mathrm{SE} < \tau), \tag{21}$$

[2]Although the convergence of WMMSE-AO with SVD combiner is hard to be mathematically proven, we had numerically validated the convergence in different aspects and all results show the fast convergence properties.

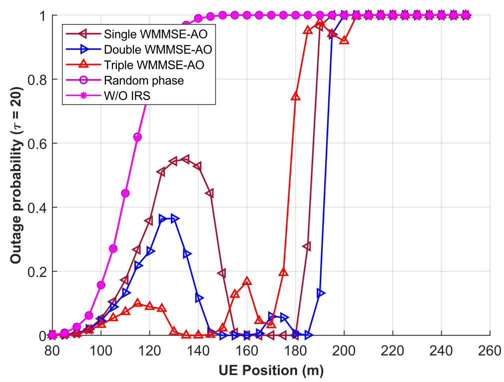

Fig. 4. Outage probability versus UE position under Scenario2.

where τ is the quality of service (QoS). In Fig. 4, with $\tau = 20$ bits/s/Hz, we observe that the outage probability can be reduced in multiple IRSs compared with single IRS, namely the total area under curves are smaller. The reason is that multiple IRSs can provide more DoF in the spatial domain to satisfy the system requirements. Therefore, multiple IRSs can achieve better coverage in the cell.

VI. Conclusion

In this paper, we transform the original spectral efficiency maximization to an WMMSE problem for single-user IRS-assisted MIMO system. Based on the WMMSE surrogate objective, we derive a low-complexity closed-form solution, which reduces the complexity by 20 times with only 1% performance degradation. Besides, its non-coupling property further enable parallel computations of IRS to achieve M times lower latency. On the other hand, to fully exploit the spatial multiplexing and increase the coverage, the proposed WMMSE/MMSE-AO can be extended to the general multi-IRS environments even the inter-IRS channels are considered. Moreover, extremely parallel update in Scenario2 without the inter-IRS channels can further shorten the optimization latency.

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